**Question 1**

N = $N\_{3} N\_{2} N\_{1} N\_{0}$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | $$N\_{3}$$ | $$N\_{2}$$ | $$N\_{1}$$ | $$N\_{0}$$ |
|  |  |  |  | $$N\_{3}$$ | $$N\_{2}$$ | $$N\_{1}$$ | $$N\_{0}$$ |
|  |  |  |  | $$N\_{0}N\_{3}$$ | $$N\_{0}N\_{2}$$ | $$N\_{0}N\_{1}$$ | $$N\_{0}N\_{0}$$ |
|  |  |  | $$N\_{1}N\_{3}$$ | $$N\_{1}N\_{2}$$ | $$N\_{1}N\_{1}$$ | $$N\_{1}N\_{0}$$ |  |
|  |  | $$N\_{2}N\_{3}$$ | $$N\_{2}N\_{2}$$ | $$N\_{2}N\_{1}$$ | $$N\_{2}N\_{0}$$ |  |  |
|  | $$N\_{3}N\_{3}$$ | $$N\_{3}N\_{2}$$ | $$N\_{3}N\_{1}$$ | $$N\_{3}N\_{0}$$ |  |  |  |
|  | $$N\_{3}N\_{2}$$ | $$N\_{1}N\_{3}$$ | $$N\_{0}N\_{3}$$ | $$N\_{0}N\_{2}$$ | $$N\_{1}N\_{0}$$ | 0 | $$N\_{0}$$ |
|  | $$N\_{3}$$ |  | $$N\_{2}N\_{1}$$ |  | $$N\_{1}$$ |  |  |
|  |  |  | $$N\_{2}$$ |  |  |  |  |

$$P\_{8}$$

$$P\_{7}$$

$$P\_{6}$$

$$P\_{5}$$

$$P\_{4}$$

$$P\_{3}$$

$$P\_{2}$$

$$P\_{1}$$

$$0$$

$$N\_{0}$$

$$N\_{3}$$

$$N\_{1}N\_{3}$$

$$N\_{3}N\_{2}$$

$$N\_{2}$$

HA

FA

FA

$$N\_{0}N\_{3}$$

$$N\_{2}N\_{1}$$

FA

$$N\_{0}N\_{2}$$

HA

$$N\_{1}N\_{0}$$

$$N\_{1}$$

HA

Area = 3 Half Adders + 3 Full Adders + AND Gate

Delay = 3$t\_{ha}$ + 3$t\_{fa}$ + AND Gate delayImplementation of 1011:

$$0$$

$$1$$

1

$$1$$

$$1$$

$$0$$

$$0$$

$$1$$

$$0$$

$$1$$

$$1$$

$$0$$

$$0$$

HA

FA

FA

$$1$$

$$0$$

FA

$$0$$

HA

$$1$$

$$1$$

HA

$$1$$

$$0$$

$$1$$

$$1$$

$$0$$

$$0$$

$$0$$

$$0$$

P = $011110010\_{2}$ = $242\_{10}$

Check $11 × 11 =121 × 2 = 242\_{10}$

**Question 2**

Let

$P\_{i}=A\_{i}⨁B\_{i}$ 

$G\_{i}= A\_{i}.B\_{i}$ 

$S\_{i}=P\_{i}⨁C\_{i}$ 

and

$$C\_{i+1}=G\_{i}+P\_{i}C\_{i}$$

|  |  |
| --- | --- |
| $$i=0$$ | $$C\_{1}=G\_{0}+P\_{0}C\_{0 }=A\_{0}B\_{0}+P\_{0}C\_{0}= G\_{0}+P\_{0}C\_{0}$$ |
| $$i=1$$ | $$C\_{2}=G\_{1}+P\_{1}C\_{1}=A\_{1}B\_{1}+P\_{1}\left(G\_{0}+P\_{0}C\_{0}\right)= G\_{1}+ P\_{1}G\_{0}+ P\_{1}P\_{0}C\_{0}$$ |
| $$i=2$$ | $$C\_{3}=G\_{2}+P\_{2}C\_{2}=G\_{2}+ P\_{2}(G\_{1}+ P\_{1}G\_{0}+ P\_{1}P\_{0}C\_{0})=G\_{2}+P\_{2}G\_{1}+ P\_{2}P\_{1}G\_{0}+ P\_{2}P\_{1}P\_{0}C\_{0}$$ |
| $$i=3$$ | $$C\_{4}=G\_{3}+P\_{3}C\_{3}=G\_{3}+ P\_{3}\left(G\_{2}+P\_{2}G\_{1}+ P\_{2}P\_{1}G\_{0}+ P\_{2}P\_{1}P\_{0}C\_{0}\right) =G\_{3}+P\_{3}G\_{2}+P\_{3}P\_{2}G\_{1}+ P\_{3}P\_{2}P\_{1}G\_{0}+ P\_{3}P\_{2}P\_{1}P\_{0}C\_{0}$$ |



$\leftarrow $**Carry Output**



$\leftarrow $**Sum Output**

Delay associated with $C\_{4}$ is the highest. The path is shown below.



$\leftarrow $**Critical path**

$$Total delay = 3+5\*\frac{1}{3}+3\*\frac{1}{3}+3\*\frac{1}{3}=8.0 τ\_{g}$$

In comparison, the 4-bit Carry Ripple Adder:

$$S\_{0}$$

$$S\_{1}$$

$$S\_{2}$$

$$S\_{3}$$

$$C\_{4}$$

$$C\_{0}$$

$$A\_{1}$$

$$A\_{3}$$

$$B\_{3}$$

$$A\_{2}$$

$$B\_{2}$$

$$B\_{1}$$

$$B\_{0}$$

$$A\_{0}$$

FA

FA

FA

FA

$$Total delay = 4 Full adder$$



$$\frac{1}{2}t\_{g}$$

$$τ\_{fa} delay = 3τ\_{g}+\frac{1}{2}τ\_{g}=3.5 τ\_{g}$$

 $Total delay = 4\*3.5=14.0 τ\_{g}$

**Question 3**

You may design a 8-state FSM with don’t cares or go for a 4-state FSM with decoder for the output which is the simplest as we follow:

State diagram

|  |  |  |
| --- | --- | --- |
| State | Next State | Output |
| $$y\_{1}$$ | $$y\_{0}$$ | $$y\_{1}^{+}$$ | $$y\_{0}^{+}$$ | $$O\_{3}$$ | $$O\_{2}$$ | $$O\_{1}$$ |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 |



From state table directly the next states and the outputs can be read as:

$$y\_{1}^{+}=y\_{1}⨁y\_{0}$$

$$y\_{0}^{+}=\overbar{y\_{0}}$$

$$O\_{3}=y\_{1}$$

$$O\_{2}=y\_{0}$$

$$O\_{1}=1$$



$$O\_{2}$$

$$O\_{3}$$

$$O\_{3}$$

$$y\_{0}$$

$$\overbar{y\_{1}}$$

$$y\_{1}$$

$$\overbar{y\_{0}}$$

Alternatively you might want to design an 8 bit FSM with don’t care states. A ,….. states 000, 010, 100, 110 never happen.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$y\_{2}$$ | $$y\_{1}$$ | $$y\_{0}$$ | $$y\_{2}^{+}$$ | $$y\_{1}^{+}$$ | $$y\_{0}^{+}$$ |
| 0 | 0 | 0 | $$x$$ | $$x$$ | $$x$$ |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | $$x$$ | $$x$$ | $$x$$ |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | $$x$$ | $$x$$ | $$x$$ |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | $$x$$ | $$x$$ | $$x$$ |
| 1 | 1 | 1 | 0 | 0 | 1 |

Giving

$$y\_{2}^{+}=y\_{1}⨁y\_{0}$$

$$y\_{1}^{+}=\overbar{y\_{1}}$$

$$y\_{0}=1$$



$\uparrow outp$uts = states

$$1$$

$$0$$

$$y\_{0}$$

$$y\_{1}$$

$$y\_{2}$$

$$O\_{1}$$

$$O\_{2}$$

$$O\_{3}$$