Register Transfer Methodology I

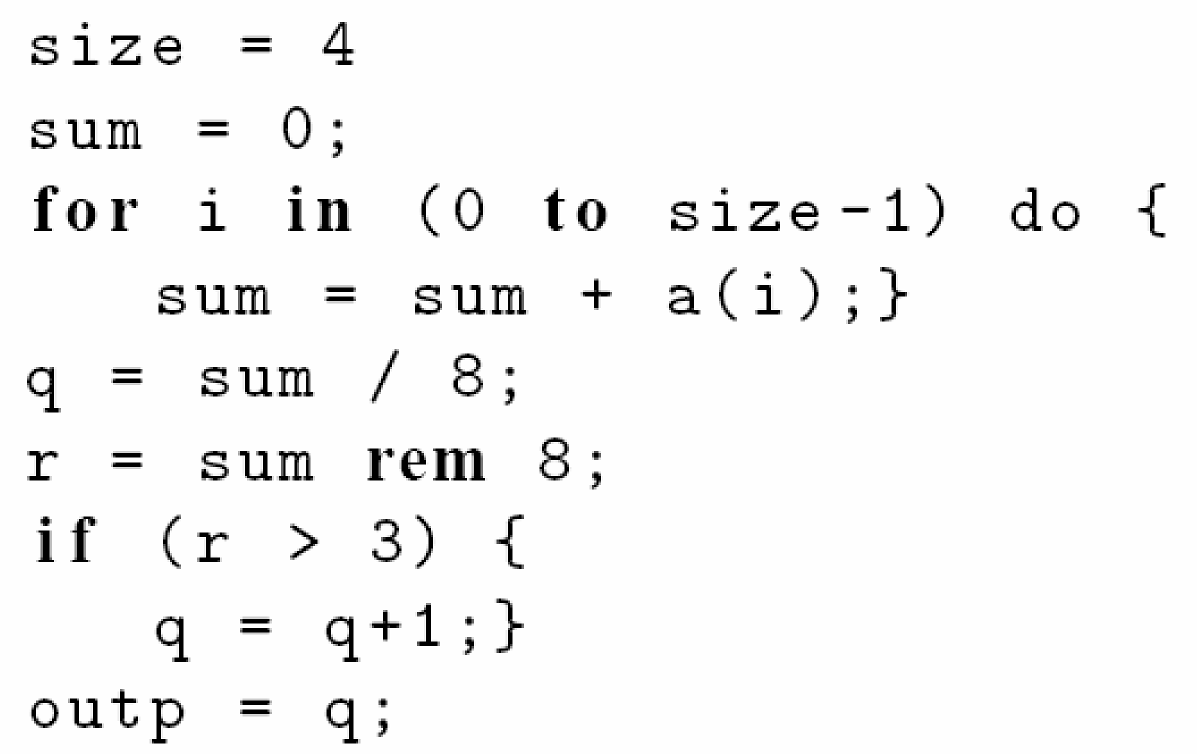
Outline

1. Introduction
2. Overview of FSMD
3. FSMD design of a repetitive-addition multiplier
4. Alternative design of a repetitive-addition multiplier
5. Timing and performance analysis of FSMD
6. Sequential add-and-shift multiplier

1. Introduction

- How to realize an algorithm in hardware?
- Two characteristics of an algorithm:
  - Use of variables (symbolic memory location) e.g., n = n + 1 in C
  - Sequential execution (execution order is important)

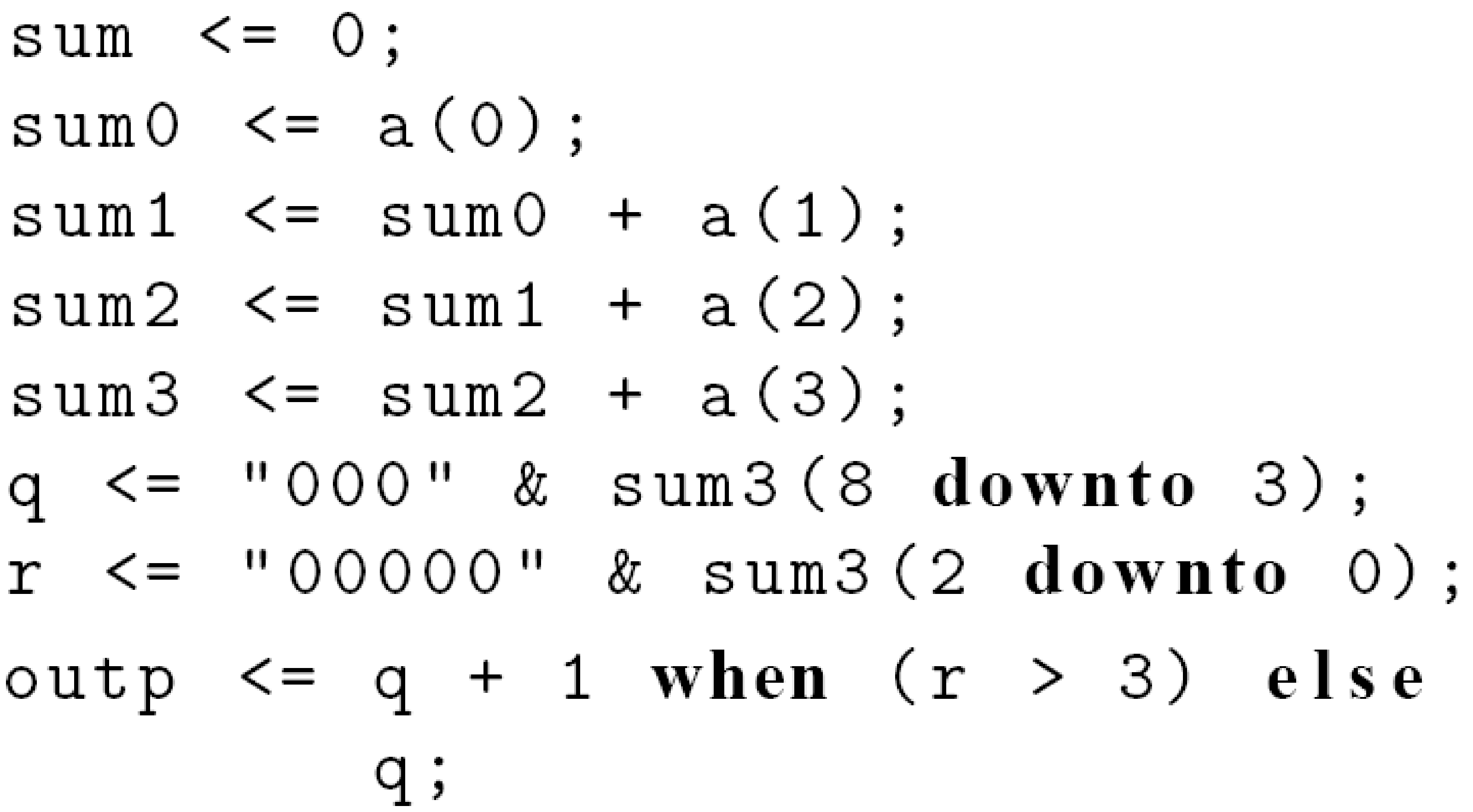
- E.g., an algorithm:
  - – Summate 4 number
  - – Divide the result by 8
  - – Round the result
- Pseudocode



```
size = 4
sum = 0;
for i in (0 to size-1) do {
   sum = sum + a(i);}
q = sum / 8;
r = sum rem 8;
if (r > 3) {
   q = q+1;}
outp = q;
```

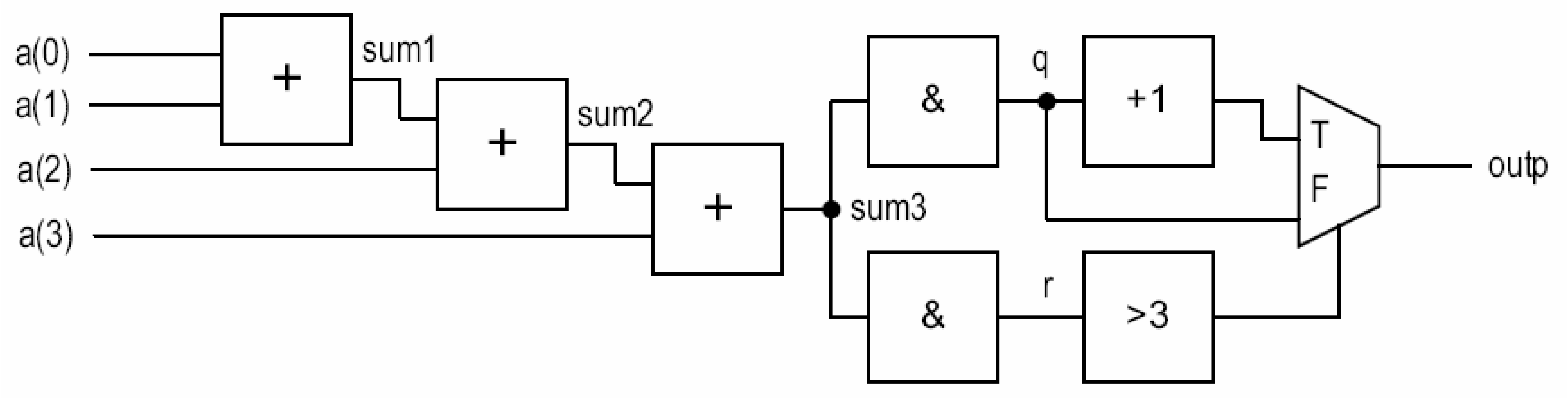
- “Dataflow” implementation in VHDL
  - Convert the algorithm in to combinational circuit
  - No memory elements
  - The sequence is embedded into the “flow of data”

- VHDL code



```
sum  <= 0;
sum0  <= a(0);
sum1  <= sum0 + a(1);
sum2  <= sum1 + a(2);
sum3  <= sum2 + a(3);
q <= "000" & sum3(8 downto 3);
r <= "00000" & sum3(2 downto 0);
outp <= q + 1 when (r > 3) else
        q;
```

- Block diagram



- Problems with dataflow implementation:
  - Can only be applied to trivial algorithm
  - Not flexible
    - Can we just share one adder in a time-multiplexing fashion to save hardware resources
    - What happen if input size is not fixed

    (i.e., size is determined by an external input)

Register Transfer Methodology

- Realized algorithm in hardware
- Use register to store intermediate data and imitate variable
- Use a datapath to realize all register operations
- Use a control path (FSM) to specify the order of register operation

- The system is specified as sequence of data manipulation/transfer among registers
- Realized by FSM with a datapath (FSMD)

2. Overview of FSMD

Basic RT operation

- Basic form:

$$r_{dest} \leftarrow f(r_{src1}, r_{src2}, \ldots, r_{srcn})$$

- Interpretation:
  - At the rising edge of the clock, the output of registers r_{src1} $r_{src2\,.}$ etc are available
  - The output are passed to a combinational circuit that performs *f( )*
  - At the next rising edge of the clock, the result is stored into r_{dest}

- E.g.,

$$r \leftarrow 1$$

$$r \leftarrow r$$

$$r0 \leftarrow r1$$

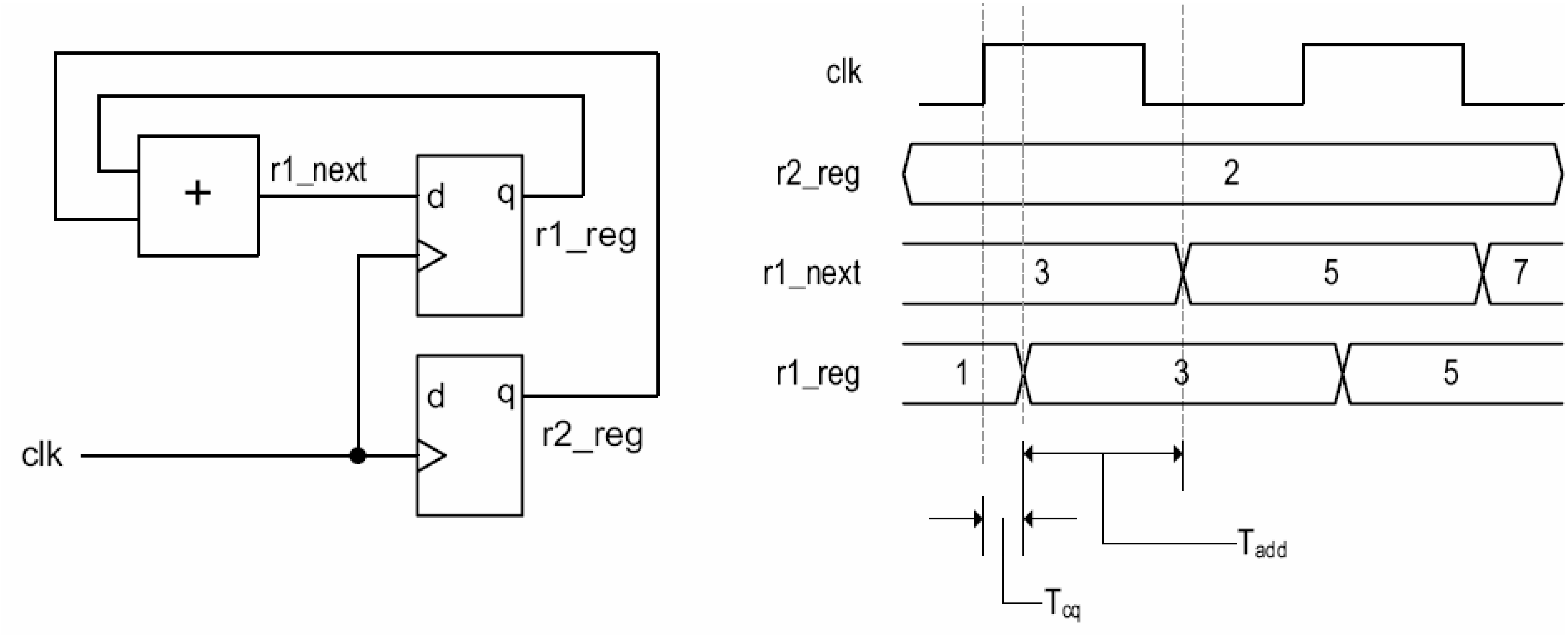
$$n \leftarrow n - 1$$

$$y \leftarrow a \oplus b \oplus c \oplus d$$

$$s \leftarrow a^2 + b^2$$

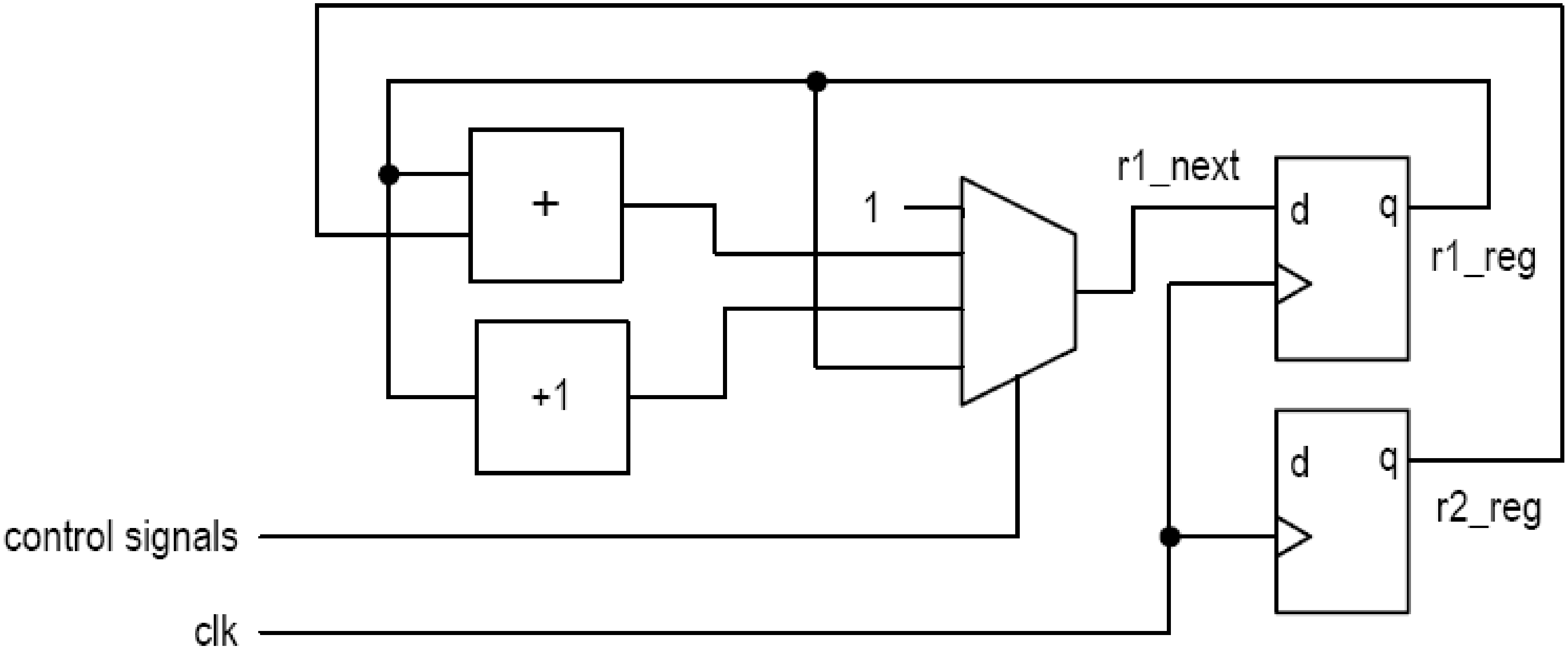
- Implementation example

$r1 \leftarrow r1+r2$



- Multiple RT operations

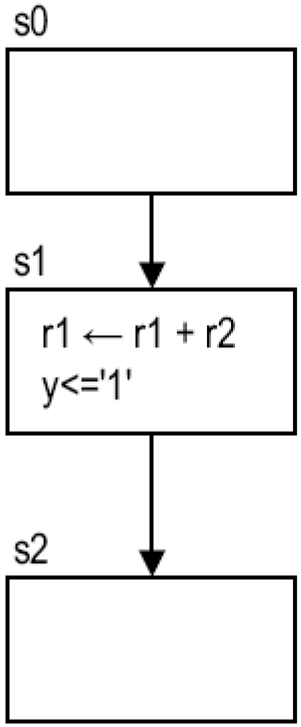
$$r1 \leftarrow 1;$$
$$r1 \leftarrow r1 + r2;$$
$$r1 \leftarrow r1 + 1;$$
$$r1 \leftarrow r1;$$



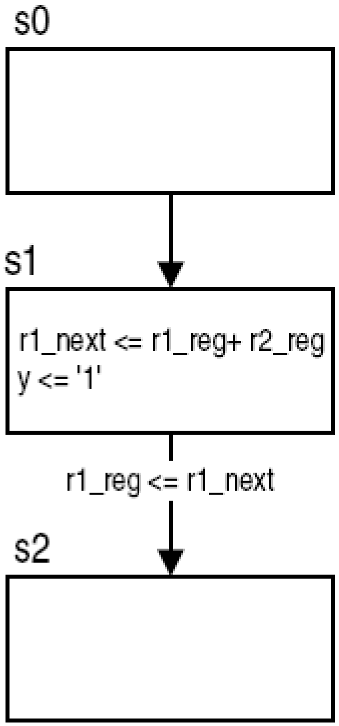
FSM as control path

- FSM is a good to control RT operation
  - State transition is on clock-by-clock basis
  - FSM can enforce order of execution
  - FSM allows branches on execution sequence
- Normally represented in an extended ASM chart known as ASMD (ASM with datapath) chart

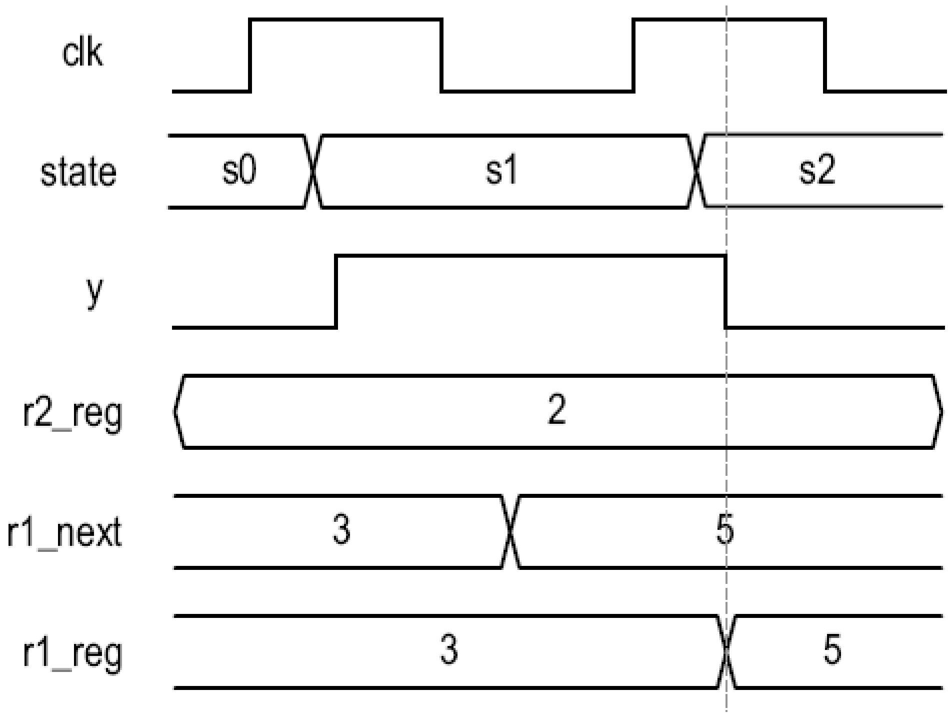
- E.g.



(a)



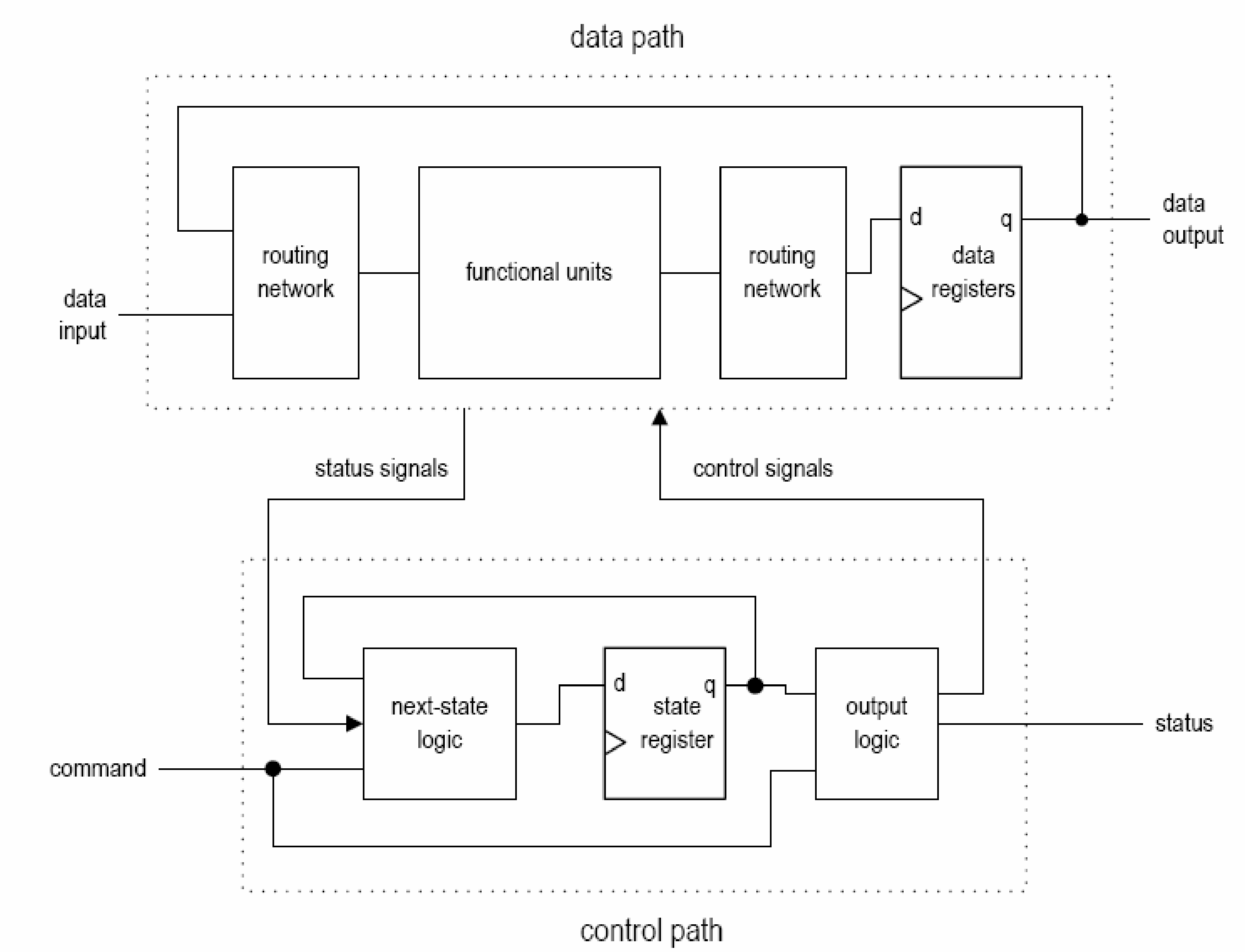
(b)



(c) Timing diagram

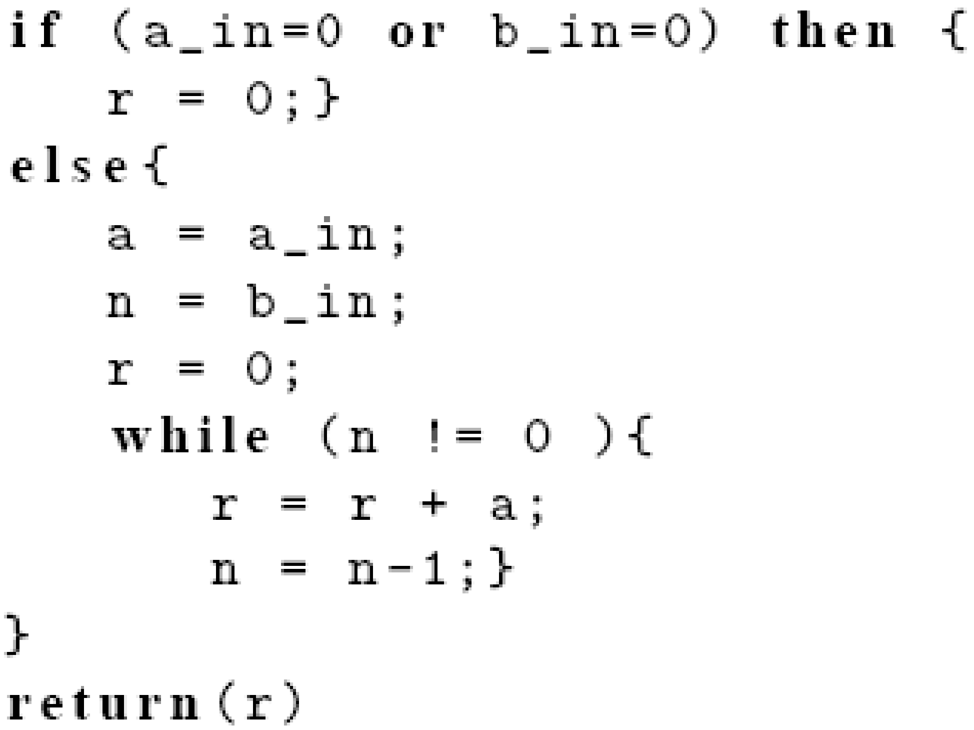
- Note: new value of r1 is only available when the FSM exits s1state

Basic Block Diagram of FSMD



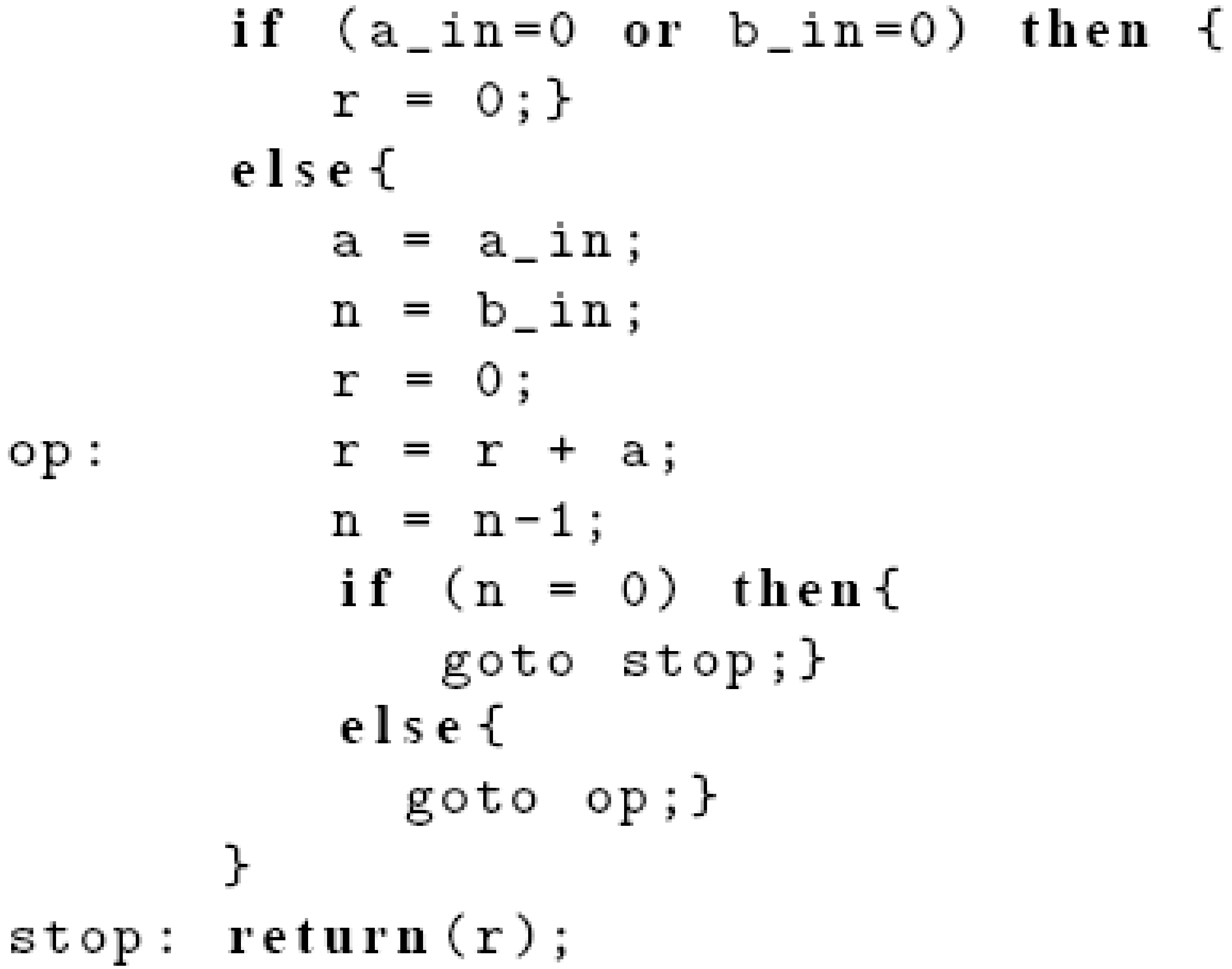
3. FSMD design example: Repetitive addition multiplier

- Basic algorithm: 7*5 = 7+7+7+7+7
- Pseudo code



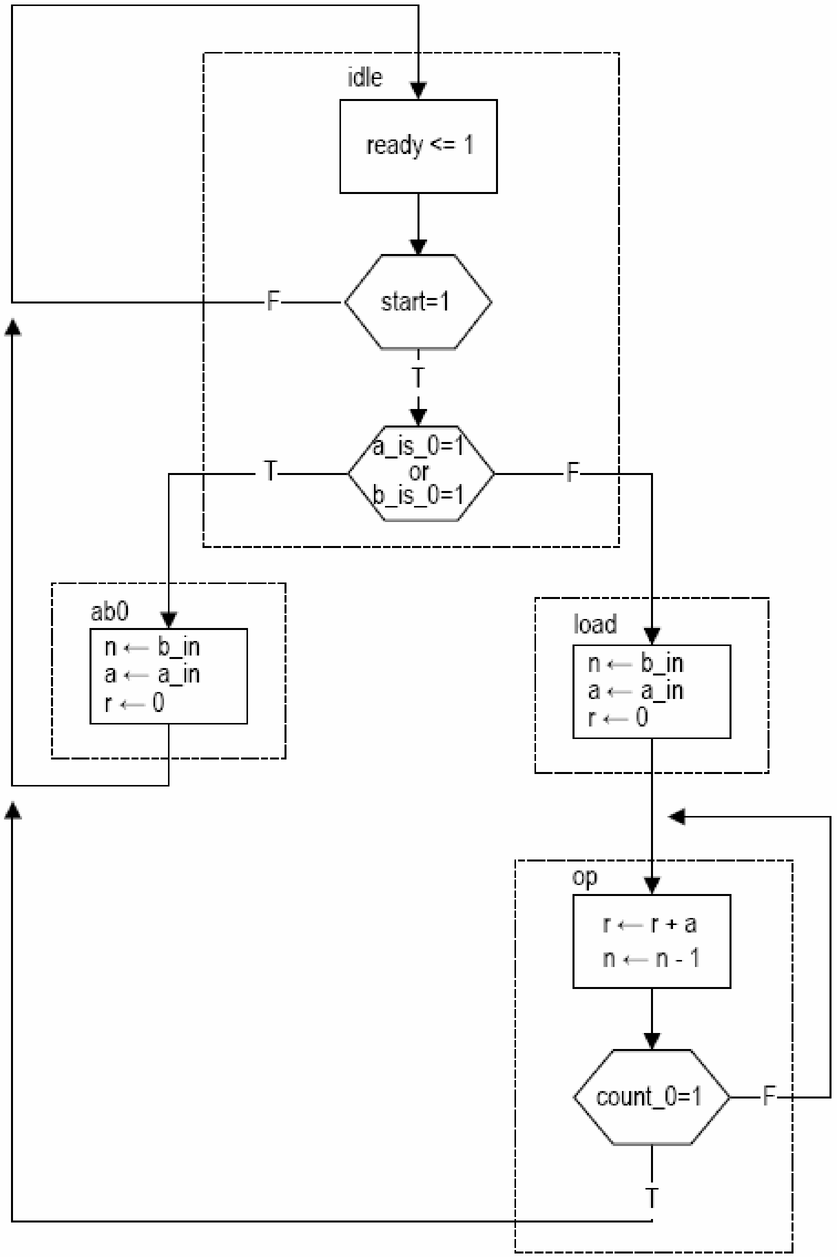
```
if (a_in=0 or b_in=0) then {
    r = 0;}
else{
    a = a_in;
    n = b_in;
    r = 0;
    while (n != 0 ){
        r = r + a;
        n = n-1;}
}
return(r)
```

- ASMD-friendly code

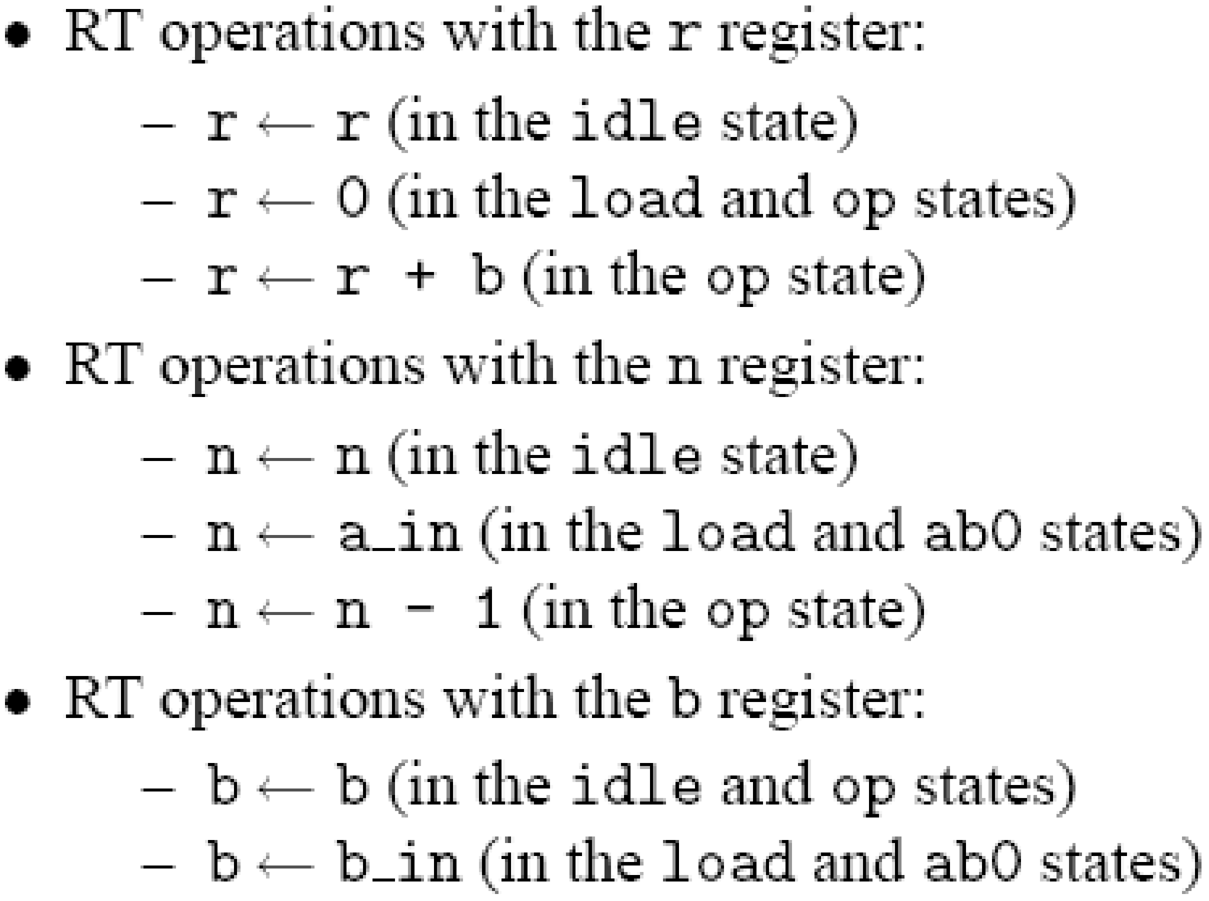


```
        if (a_in=0 or b_in=0) then {
           r = 0;}
        else{
           a = a_in;
           n = b_in;
           r = 0;
op:        r = r + a;
           n = n-1;
           if (n = 0) then{
              goto stop;}
           else{
             goto op;}
        }
stop:   return(r);
```

- Input:
  - a_in, b_in: 8-bit unsigned
  - clk, reset
  - start: command
- Output:
  - r: 16-bit unsigned
  - ready: status
- ASMD chart
  - Default RT operation: keep the previous value
  - Note the parallel execution in op state

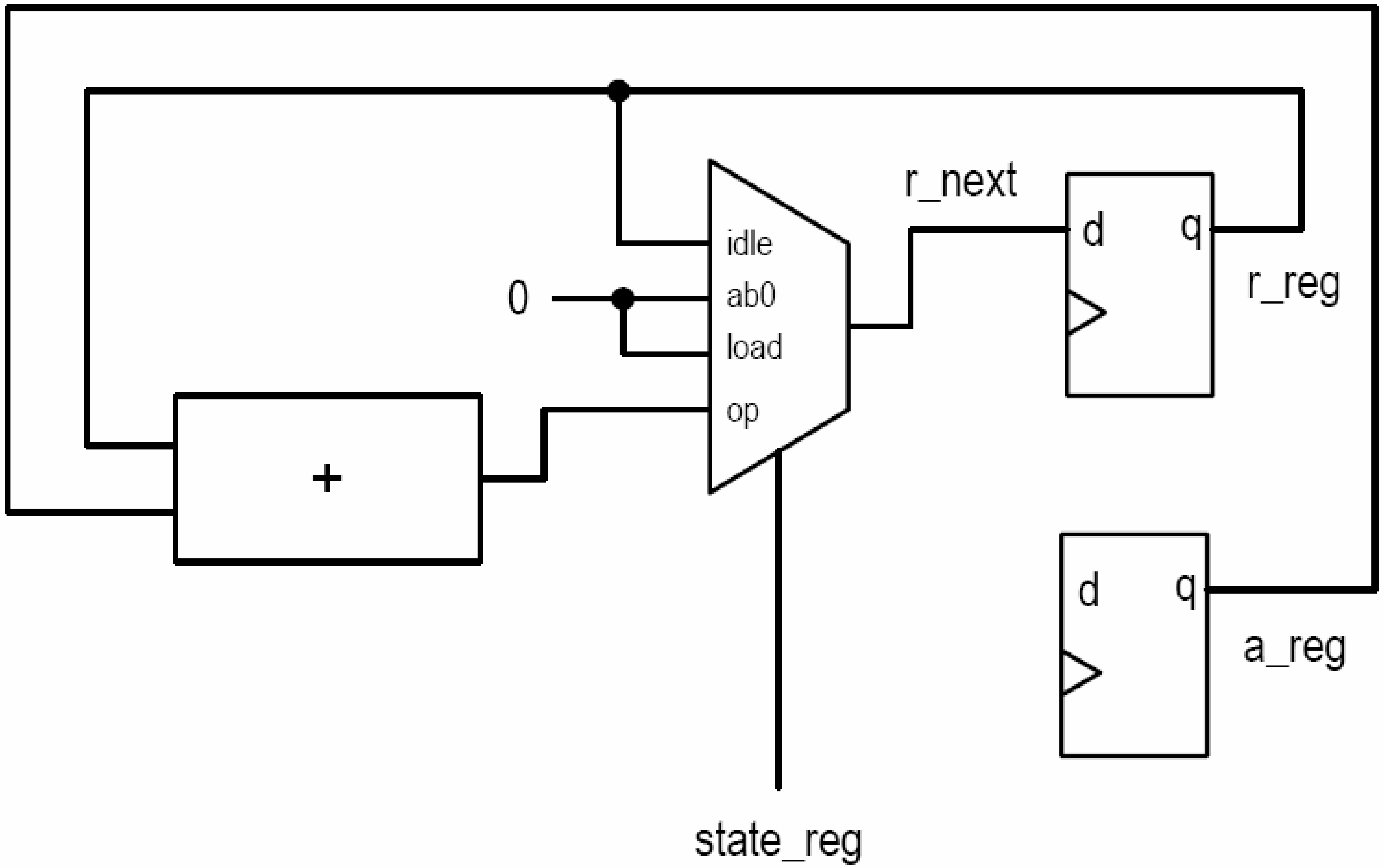


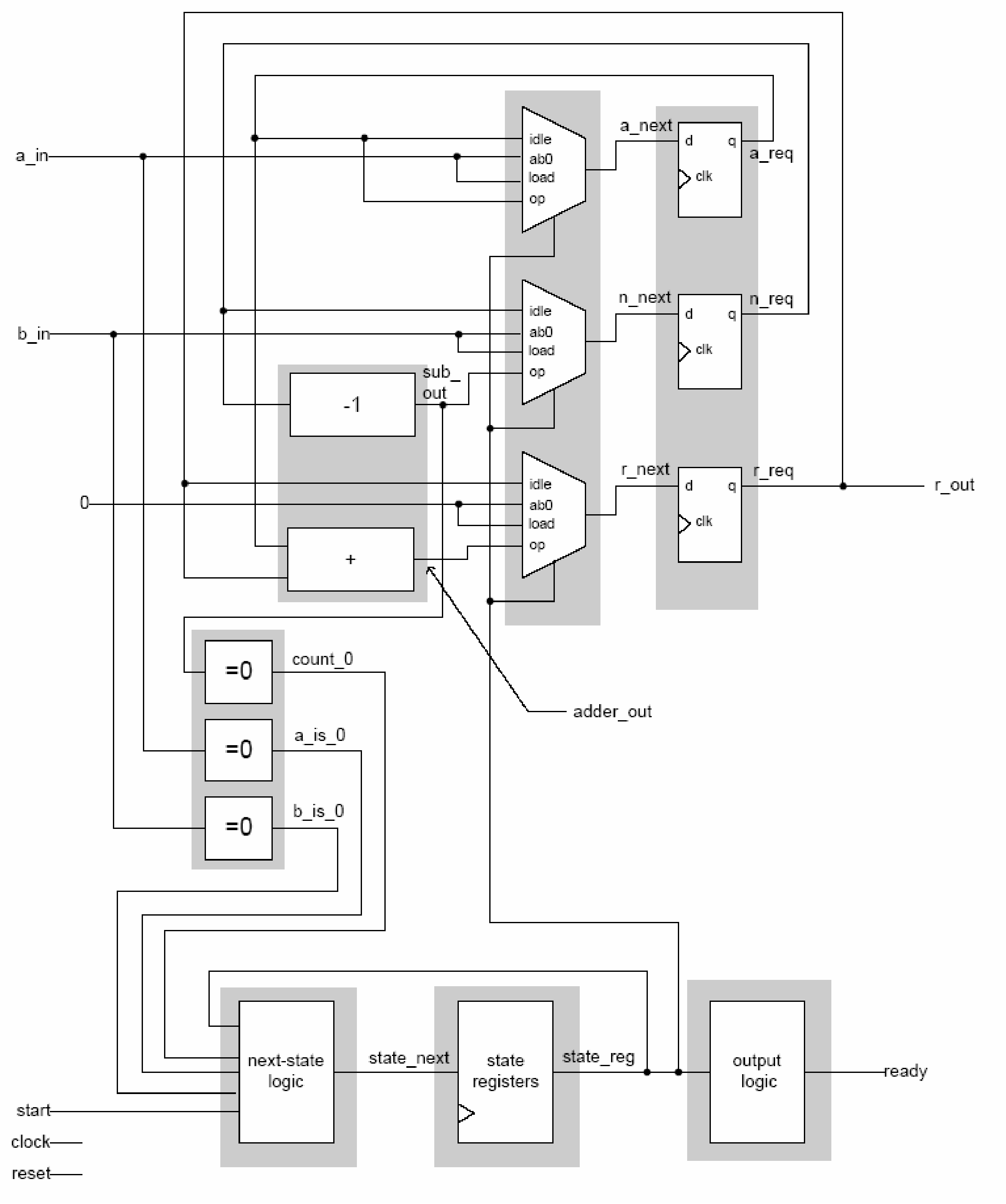
- Construction of the data path
  - List all RT operations
  - Group RT operation according to the destination register
  - Add combinational circuit/mux
  - Add status circuits
- E.g



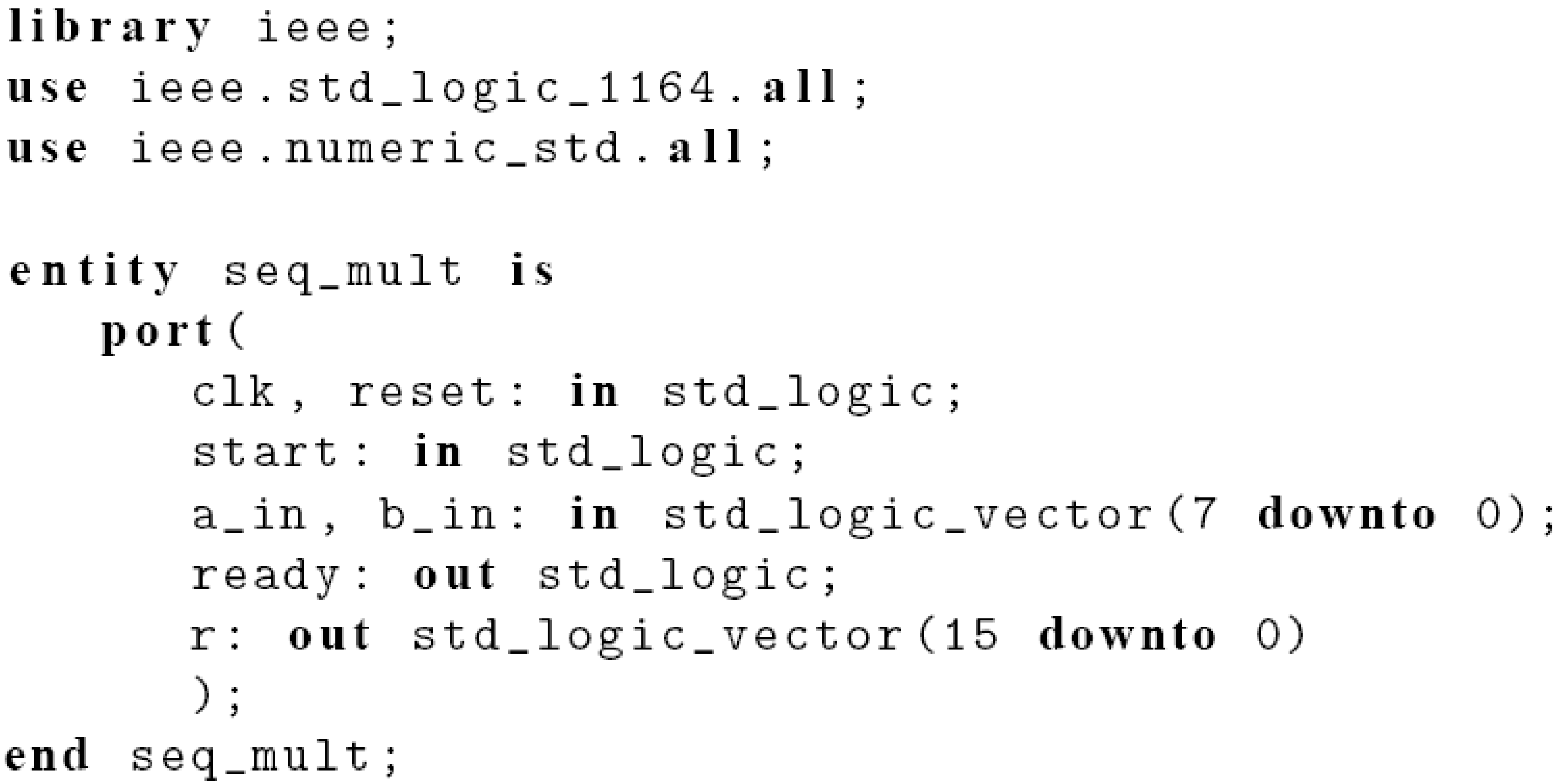
- RT operations with the `r` register:
  - `r` ← `r` (in the `idle` state)
  - `r` ← `0` (in the `load` and `op` states)
  - `r` ← `r + b` (in the `op` state)
- RT operations with the `n` register:
  - `n` ← `n` (in the `idle` state)
  - `n` ← `a_in` (in the `load` and `ab0` states)
  - `n` ← `n - 1` (in the `op` state)
- RT operations with the `b` register:
  - `b` ← `b` (in the `idle` and `op` states)
  - `b` ← `b_in` (in the `load` and `ab0` states)

- E.g., Circuit associated with r register

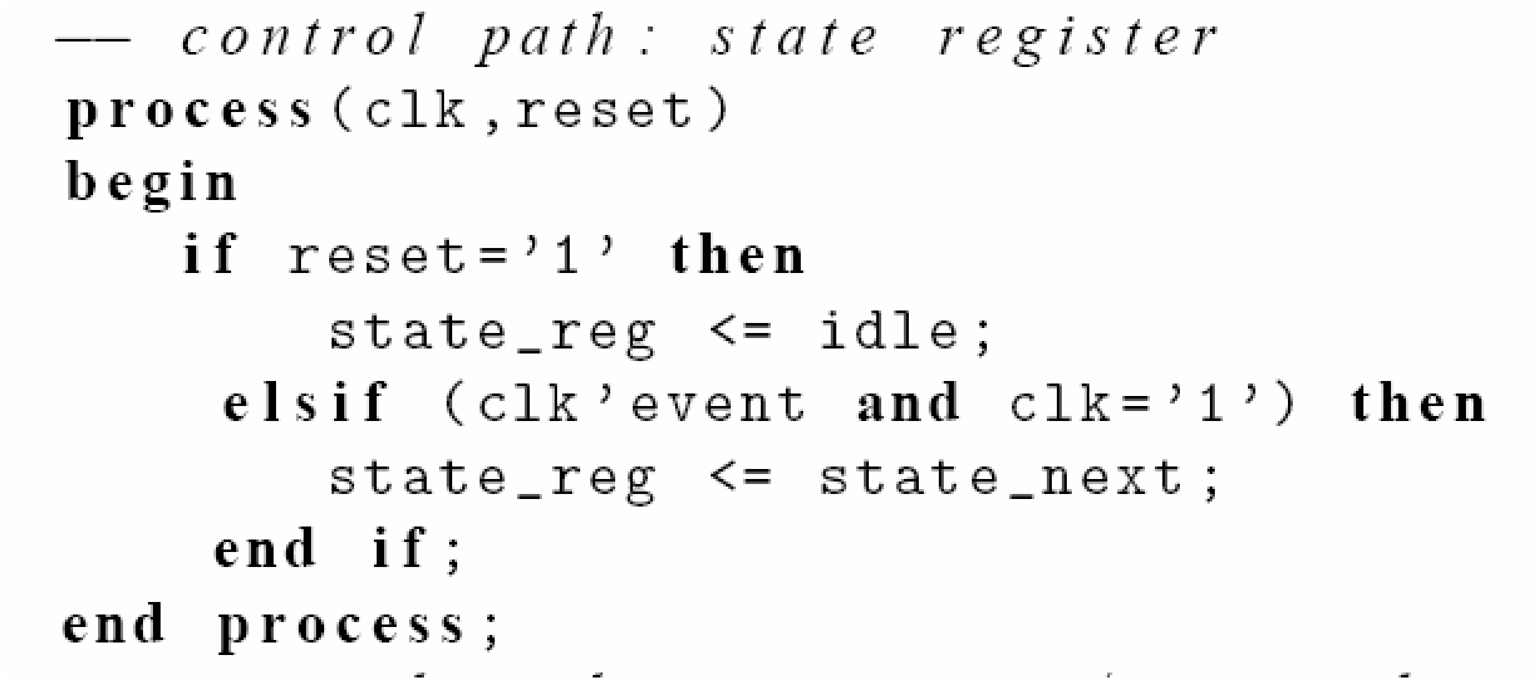



a_in
b_in
0
idle
ab0
load
op
a_next
n_next
r_next
d
q
clk
a_req
n_req
r_req
r_out
-1
+
sub_
out
adder_out
=0
count_0
a_is_0
b_is_0
next-state logic
state_next
state registers
state_reg
output logic
ready
start
clock
reset

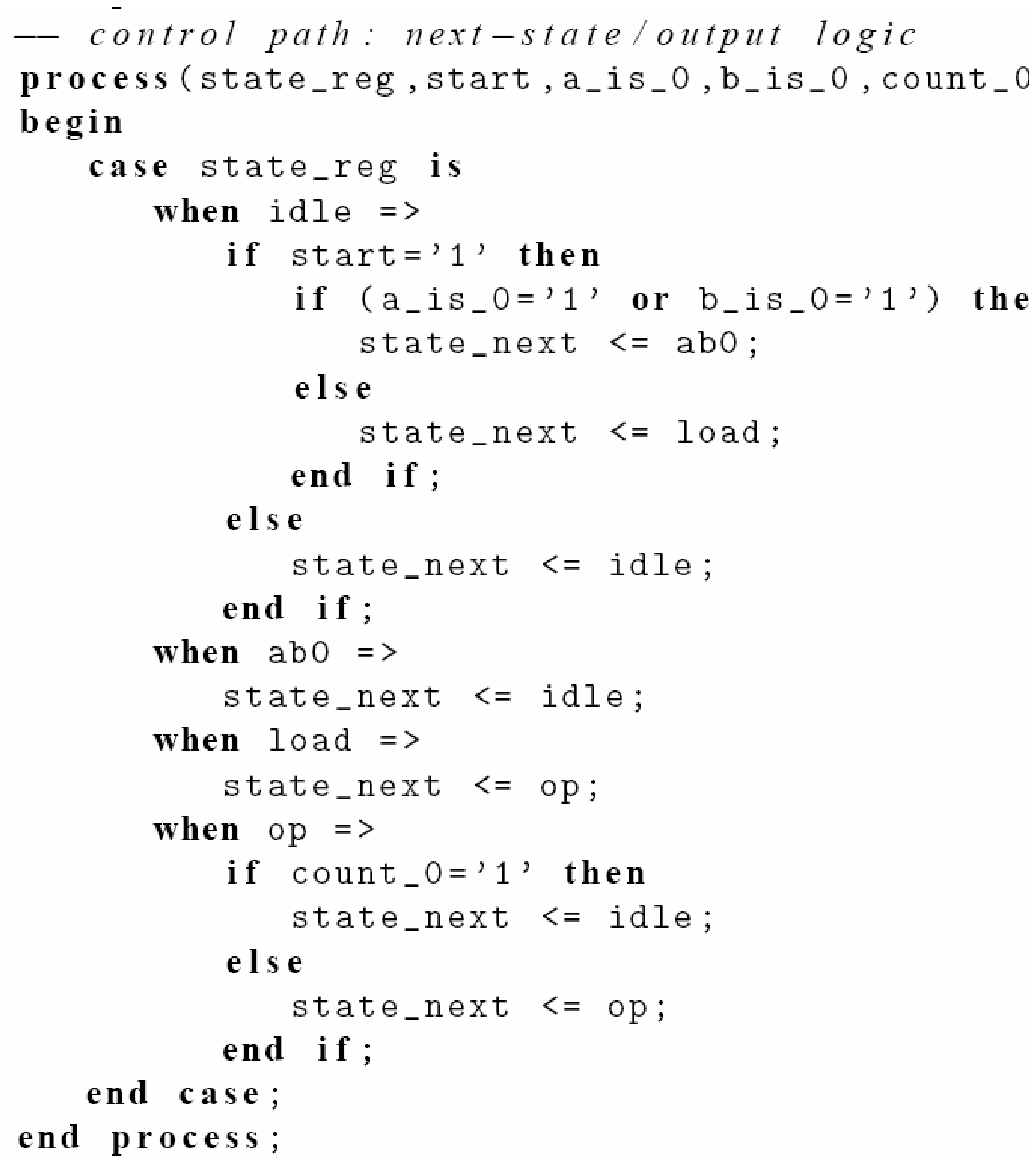
- VHDL code: follow the block diagram



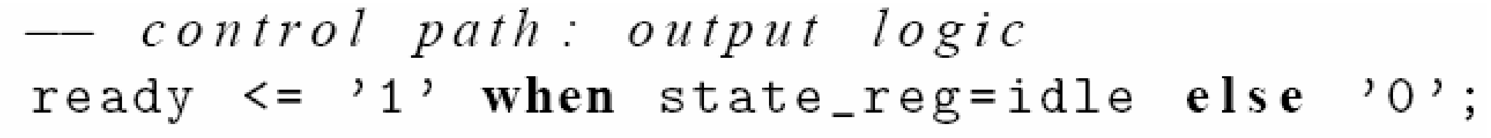
```
library ieee;
use ieee.std_logic_1164.all;
use ieee.numeric_std.all;

entity seq_mult is
   port(
      clk, reset: in std_logic;
      start: in std_logic;
      a_in, b_in: in std_logic_vector(7 downto 0);
      ready: out std_logic;
      r: out std_logic_vector(15 downto 0)
      );
end seq_mult;
```



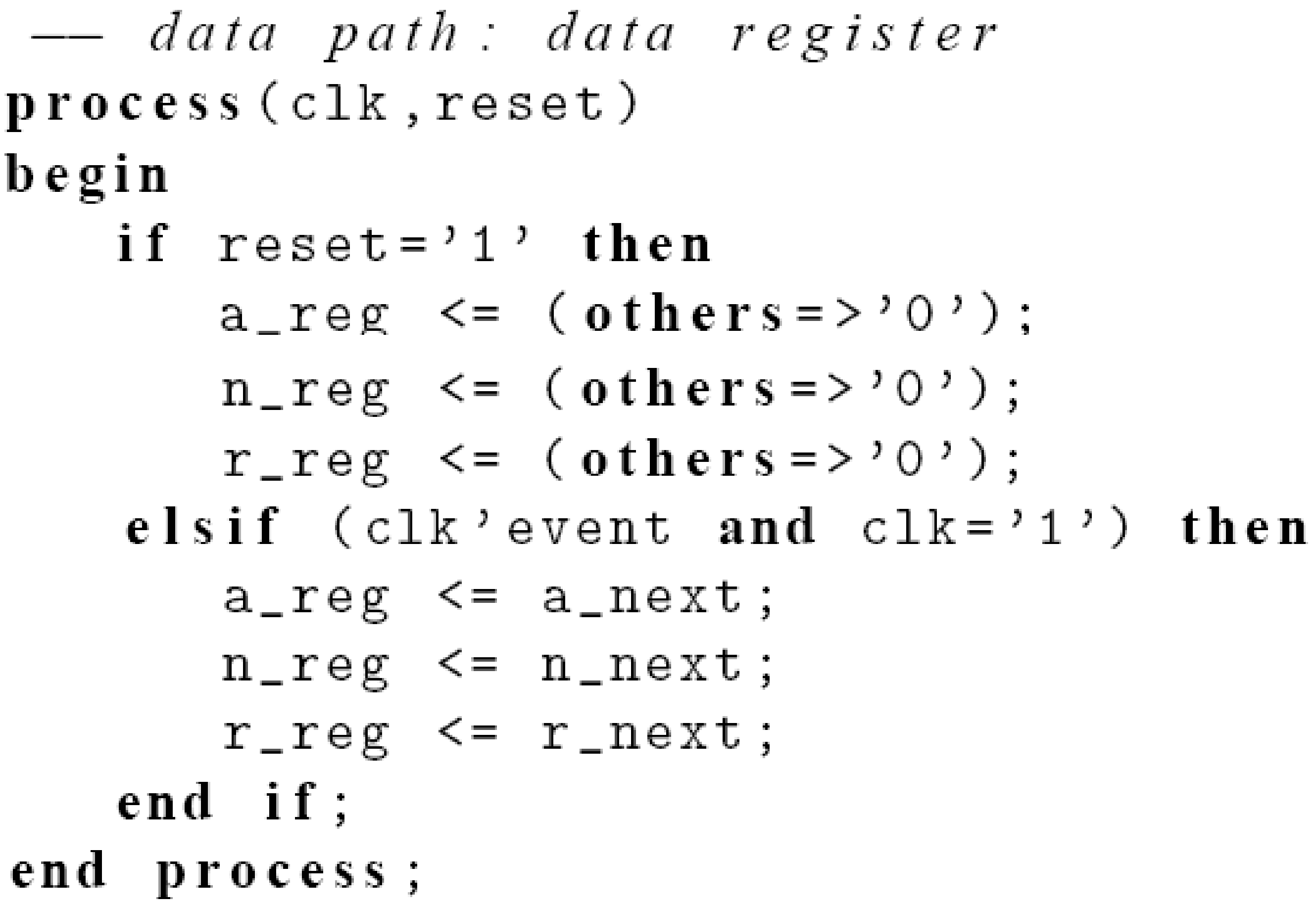
```
-- control path: state register
process(clk,reset)
begin
   if reset='1' then
      state_reg <= idle;
    elsif (clk'event and clk='1') then
      state_reg <= state_next;
    end if;
end process;
```



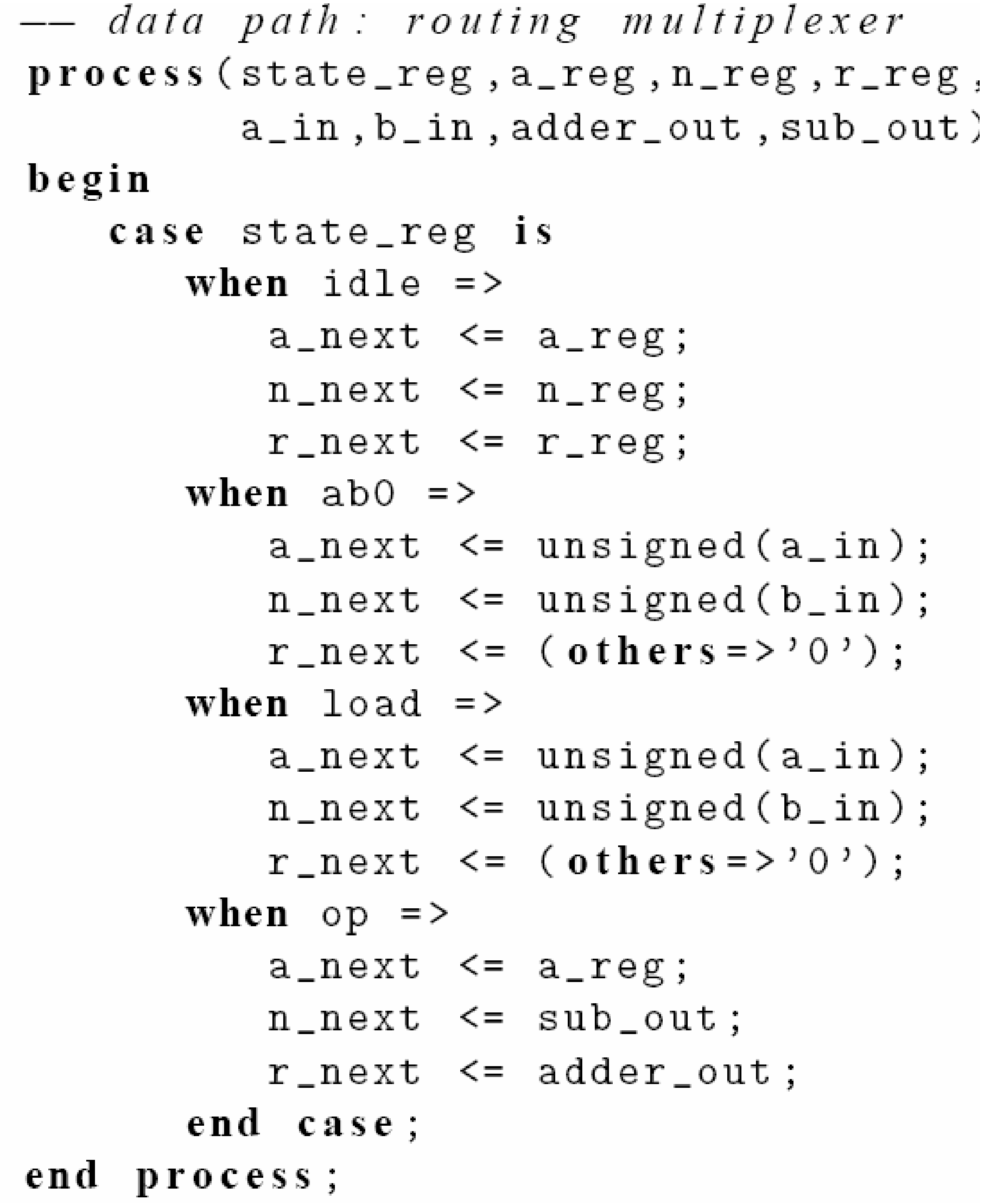
```
-- control path: next-state/output logic
process(state_reg,start,a_is_0,b_is_0,count_0
begin
   case state_reg is
      when idle =>
         if start='1' then
            if (a_is_0='1' or b_is_0='1') the
               state_next <= ab0;
            else
               state_next <= load;
            end if;
         else
            state_next <= idle;
         end if;
      when ab0 =>
         state_next <= idle;
      when load =>
         state_next <= op;
      when op =>
         if count_0='1' then
            state_next <= idle;
         else
            state_next <= op;
         end if;
   end case;
end process;
```



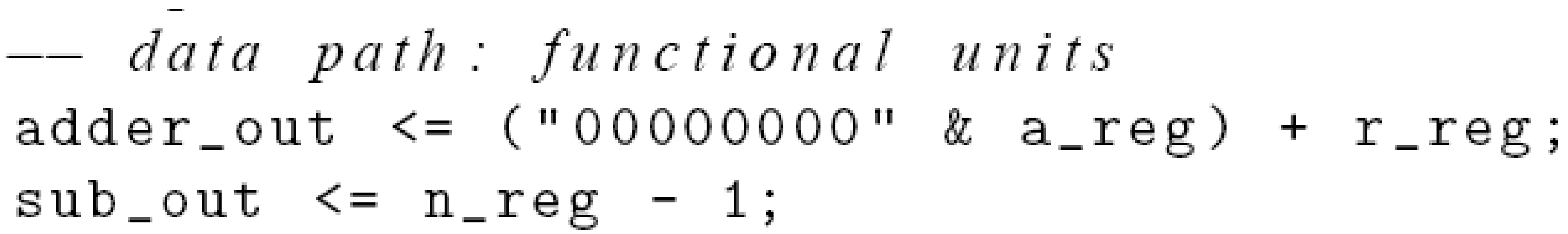
```
— control path: output logic
ready <= ’1’ when state_reg=idle else ’0’;
```



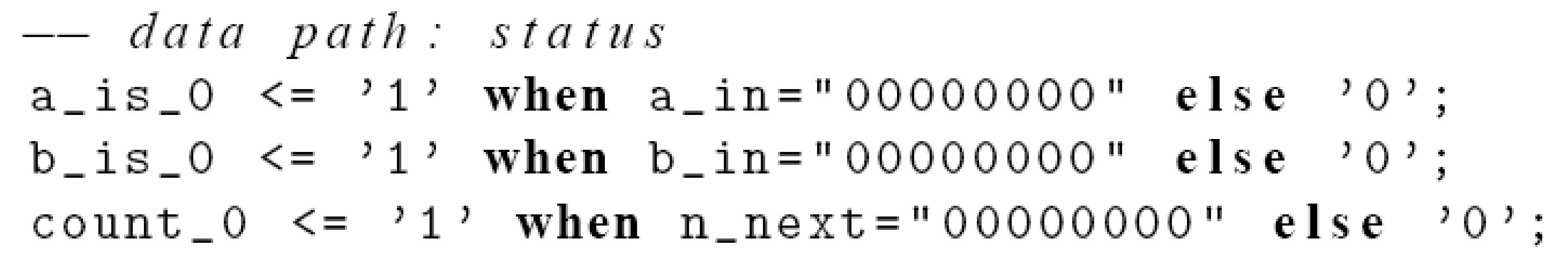
```
 — data path: data register
process(clk,reset)
begin
   if reset=’1’ then
      a_reg <= (others=>’0’);
      n_reg <= (others=>’0’);
      r_reg <= (others=>’0’);
   elsif (clk’event and clk=’1’) then
      a_reg <= a_next;
      n_reg <= n_next;
      r_reg <= r_next;
   end if;
end process;
```



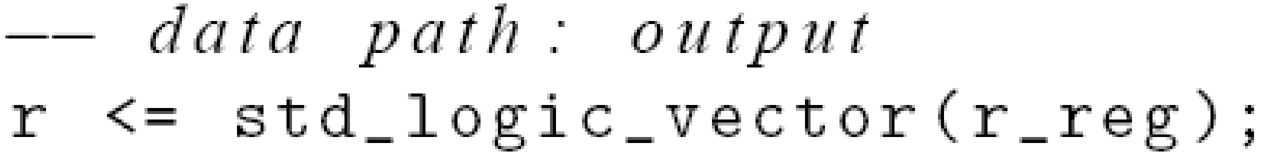
```
-- data path: routing multiplexer
process(state_reg,a_reg,n_reg,r_reg,
        a_in,b_in,adder_out,sub_out)
begin
   case state_reg is
      when idle =>
         a_next <= a_reg;
         n_next <= n_reg;
         r_next <= r_reg;
      when ab0 =>
         a_next <= unsigned(a_in);
         n_next <= unsigned(b_in);
         r_next <= (others=>'0');
      when load =>
         a_next <= unsigned(a_in);
         n_next <= unsigned(b_in);
         r_next <= (others=>'0');
      when op =>
         a_next <= a_reg;
         n_next <= sub_out;
         r_next <= adder_out;
      end case;
end process;
```



```
-- data path: functional units
adder_out <= ("00000000" & a_reg) + r_reg;
sub_out <= n_reg - 1;
```

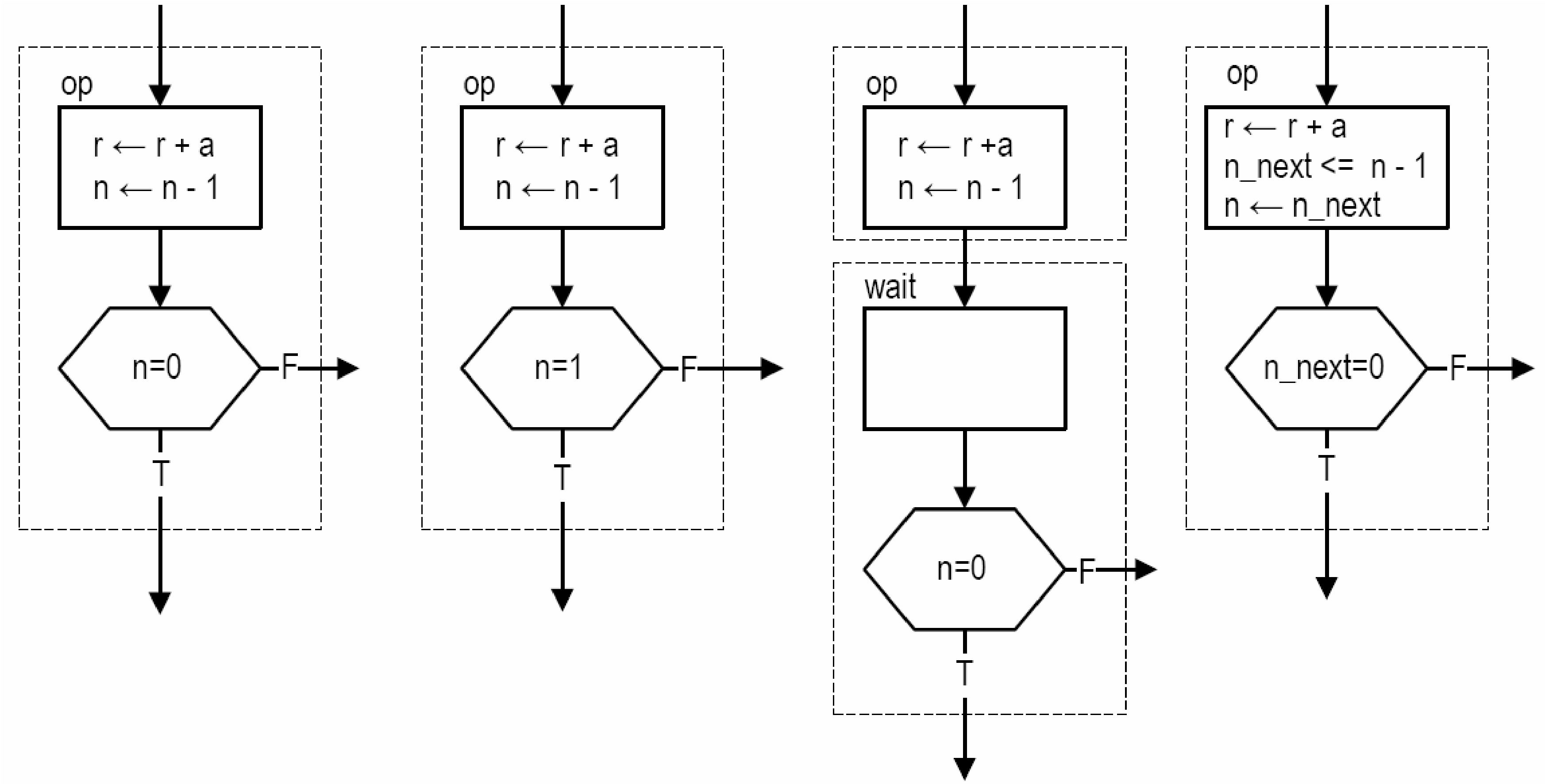


```
-- data path: status
a_is_0 <= '1' when a_in="00000000" else '0';
b_is_0 <= '1' when b_in="00000000" else '0';
count_0 <= '1' when n_next="00000000" else '0';
```



```
-- data path: output
r <= std_logic_vector(r_reg);
```

- Use of register in decision box
  - Register is updated when the FSM exits current state
  - How to represent count_0='1' using register?



- Other VHDL coding styles:
  - Various code segments can be combined
  - Should always separate registers from combinational logic
  - May be a good idea to isolate the main functional units

- E.g., 2-segment code

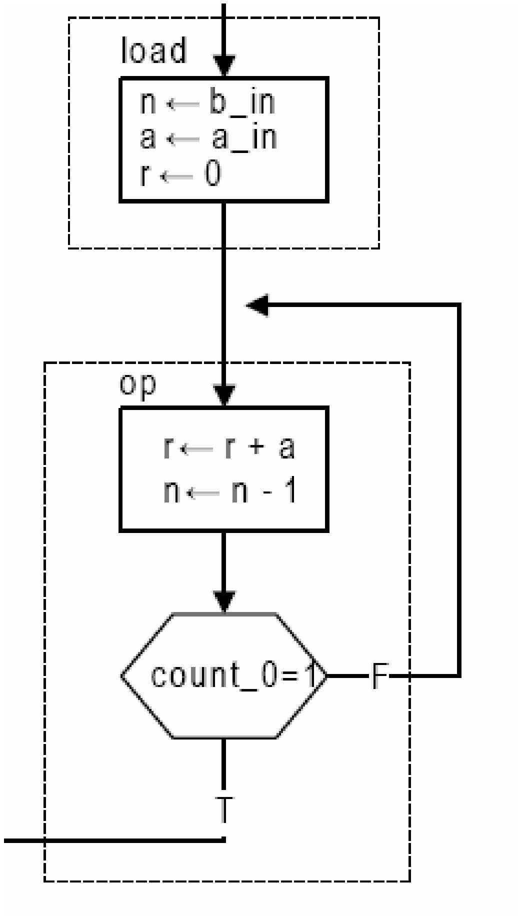
```
-- state and data register
process(clk,reset)
begin
   if reset='1' then
      state_reg <= idle;
      a_reg <= (others=>'0');
      n_reg <= (others=>'0');
      r_reg <= (others=>'0');
   elsif (clk'event and clk='1') then
      state_reg <= state_next;
      a_reg <= a_next;
      n_reg <= n_next;
      r_reg <= r_next;
   end if;
end process;
```

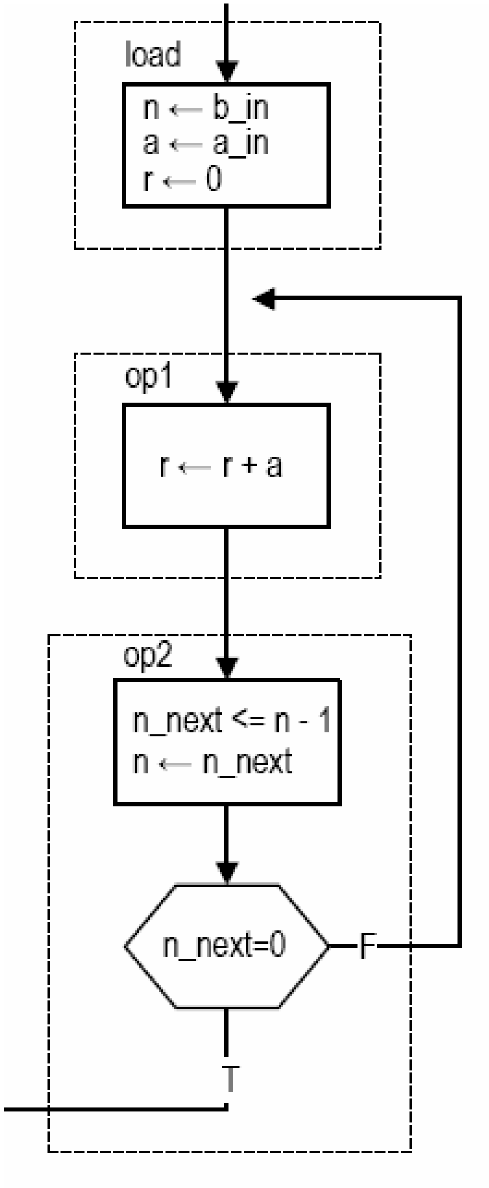
```
-- combinational circuit
process(start,state_reg,a_reg,n_reg,r_reg,
        a_in,b_in,n_next)
begin
   -- default value
   a_next <= a_reg;
   n_next <= n_reg;
   r_next <= r_reg;
   ready <=’0’;
   case state_reg is
      when idle =>
         if start=’1’ then
            if (a_in="00000000" or b_in="00000000") then
               state_next <= ab0;
            else
               state_next <= load;
            end if;
         else
              state_next <= idle;
         end if;
         ready <=’1’;
```

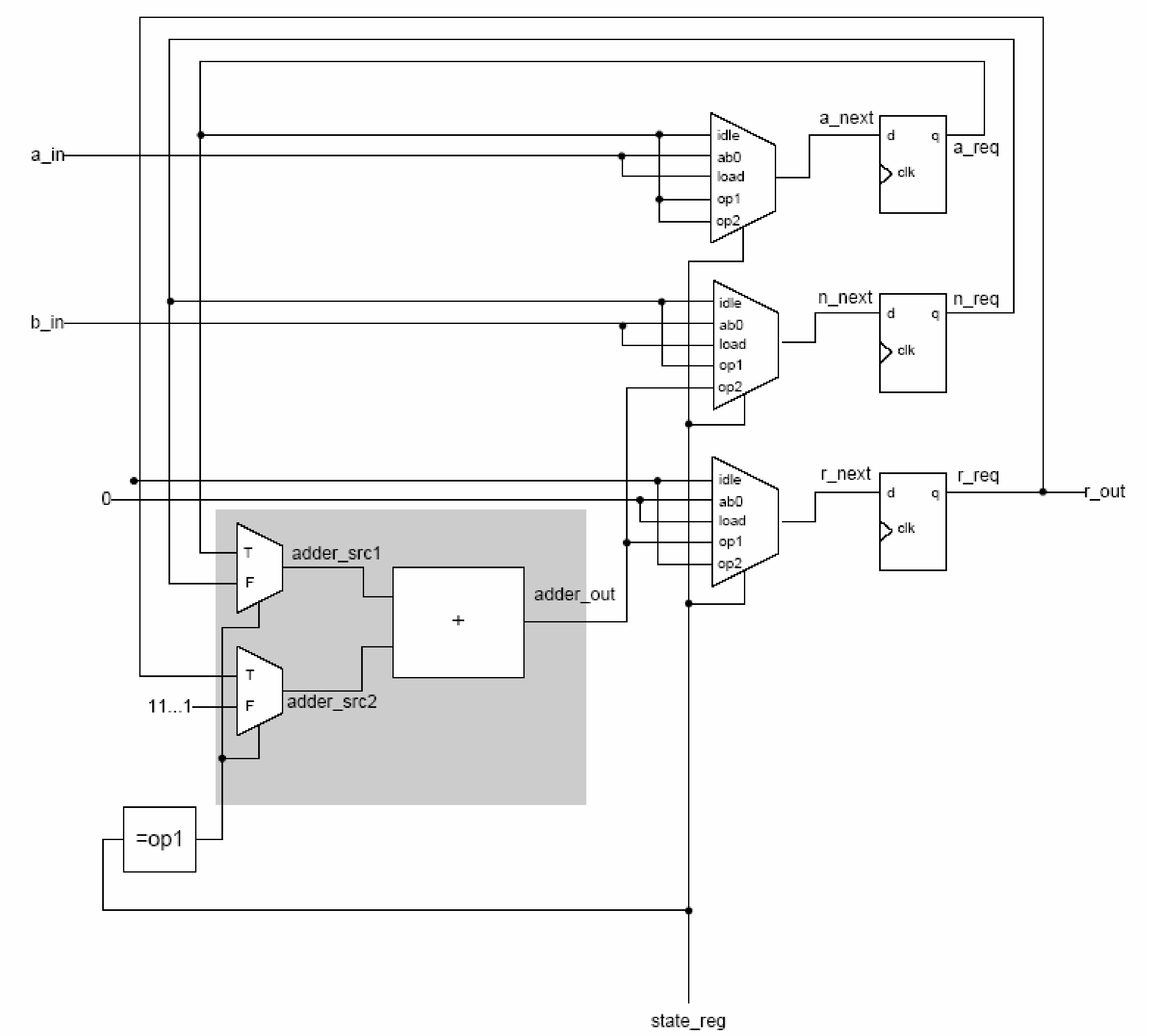
```
    when ab0 =>
      a_next <= unsigned(a_in);
      n_next <= unsigned(b_in);
      r_next <= (others=>'0');
      state_next <= idle;
    when load =>
      a_next <= unsigned(a_in);
      n_next <= unsigned(b_in);
      r_next <= (others=>'0');
      state_next <= op;
      ready <='0';
    when op =>
      n_next <= n_reg - 1;
      r_next <= ("00000000" & a_reg) + r_reg;
      if (n_next="00000000") then
        state_next <= idle;
      else
        state_next <= op;
      end if;
      ready <='0';
  end case;
end process;
r <= std_logic_vector(r_reg);
```

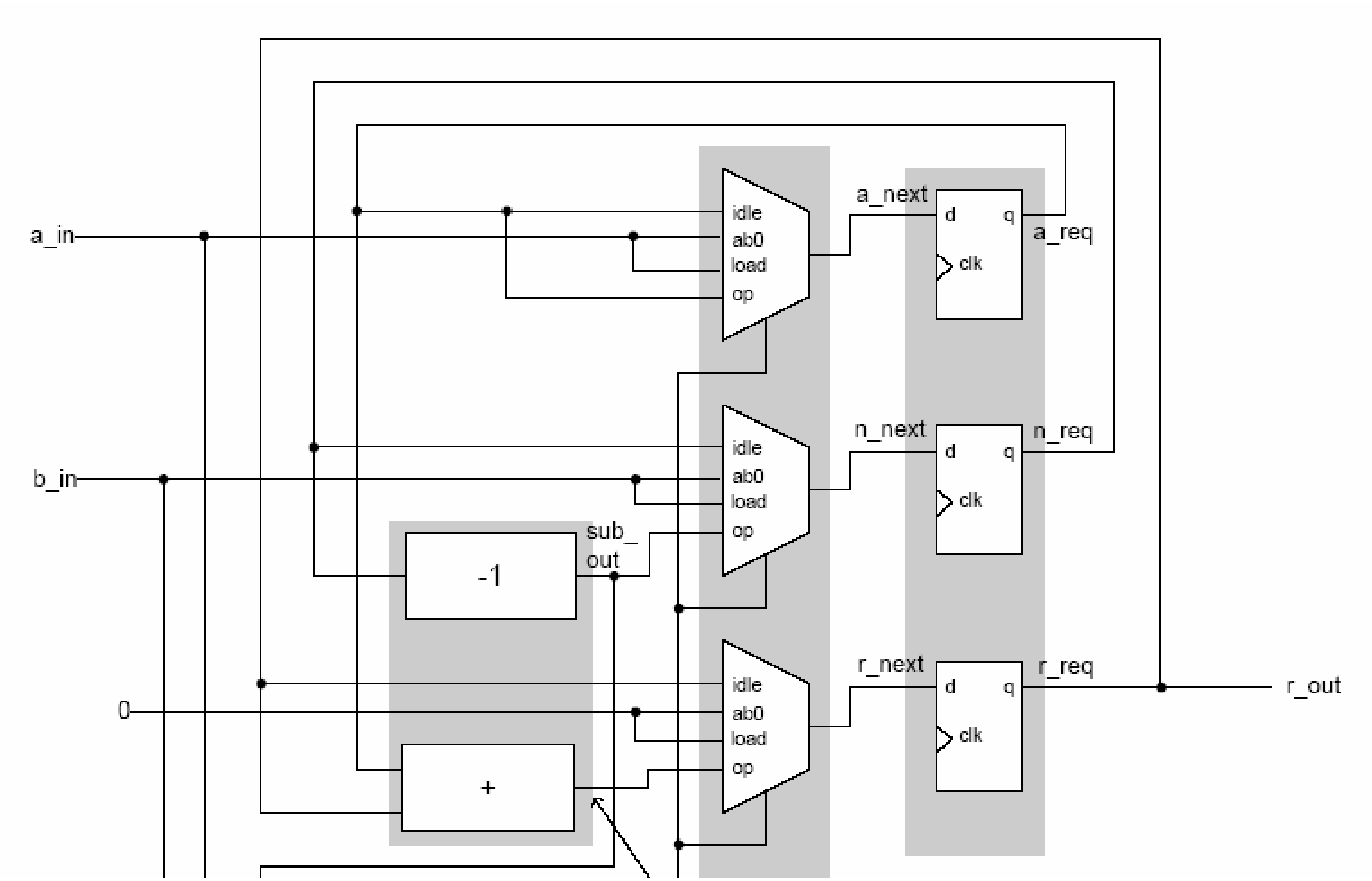
4. Alternative design of a repetitive-addition multiplier

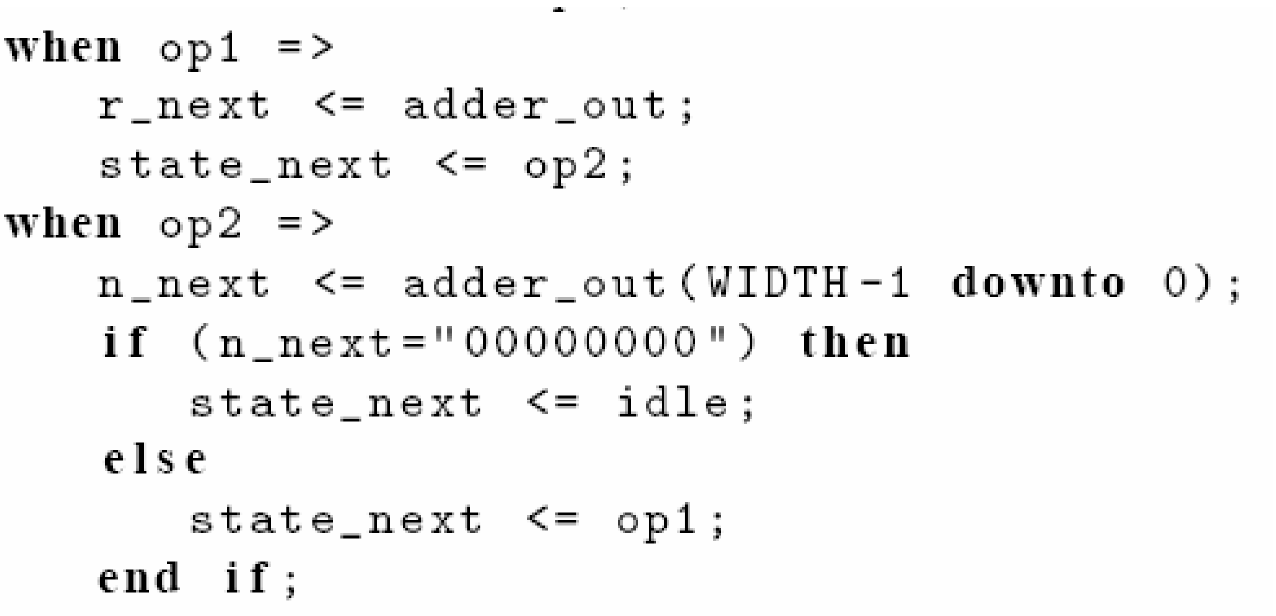
- Resource sharing
  - Hardware can be shared in a time-multiplexing fashion
  - Assign the operation in different states
  - Most complex circuits in the FSMD design is normally the functional units of the datapath
- Sharing in repetitive addition multiplier
  - Addition and decrementing
  - The same adder can be used in 2 states


load
n ← b_in
a ← a_in
r ← 0
op
r ← r + a
n ← n - 1
count_0=1
F
T

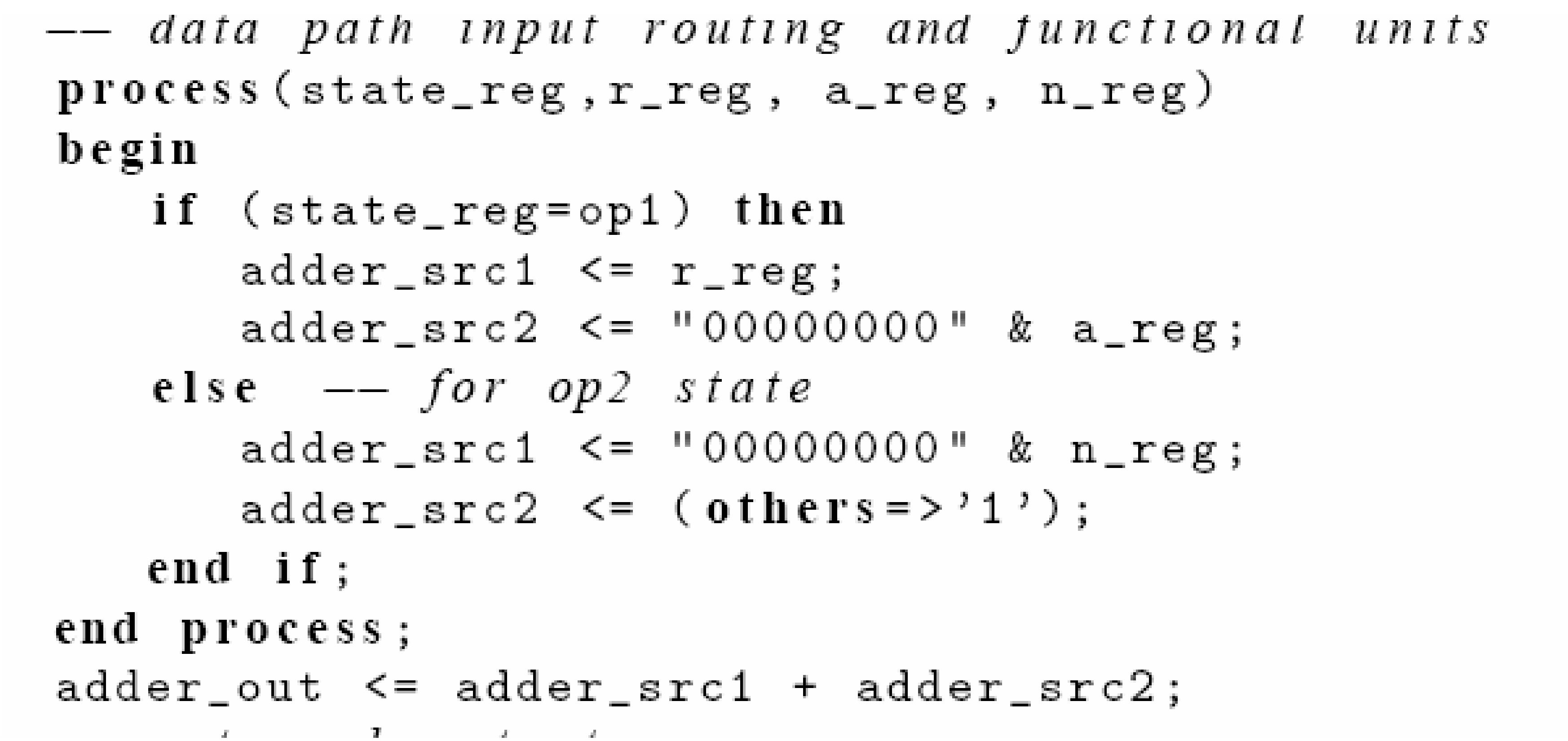

load
n ← b_in
a ← a_in
r ← 0
op1
r ← r + a
op2
n_next <= n - 1
n ← n_next
n_next=0
F
T


a_in
b_in
0
11...1
idle
ab0
load
op1
op2
a_next
n_next
r_next
d
q
clk
a_req
n_req
r_req
r_out
T
F
adder_src1
adder_src2
+
adder_out
=op1
state_reg


a_in
b_in
0
idle
ab0
load
op
a_next
n_next
r_next
d
q
clk
a_req
n_req
r_req
r_out
-1
+
sub_
out

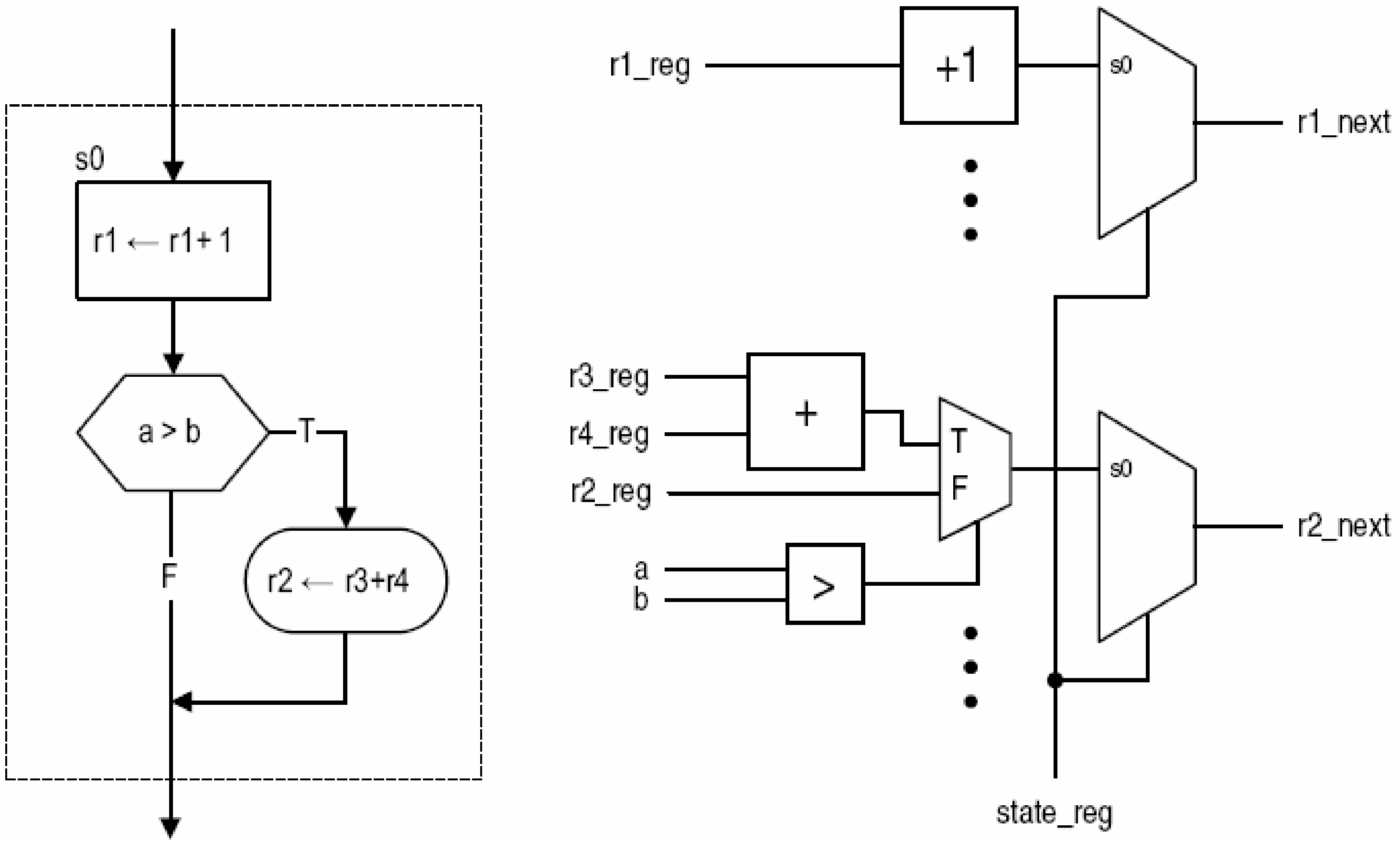


```
when op1 =>
   r_next <= adder_out;
   state_next <= op2;
when op2 =>
   n_next <= adder_out(WIDTH-1 downto 0);
   if (n_next="00000000") then
      state_next <= idle;
   else
      state_next <= op1;
   end if;
```



```
-- data path input routing and functional units
process(state_reg,r_reg, a_reg, n_reg)
begin
   if (state_reg=op1) then
      adder_src1 <= r_reg;
      adder_src2 <= "00000000" & a_reg;
   else  -- for op2 state
      adder_src1 <= "00000000" & n_reg;
      adder_src2 <= (others=>'1');
   end if;
end process;
adder_out <= adder_src1 + adder_src2;
```

- Mealy-controlled operation
  - Control signals is edge-sensitive
  - Mealy output is faster and requires fewer states
  - E.g.,

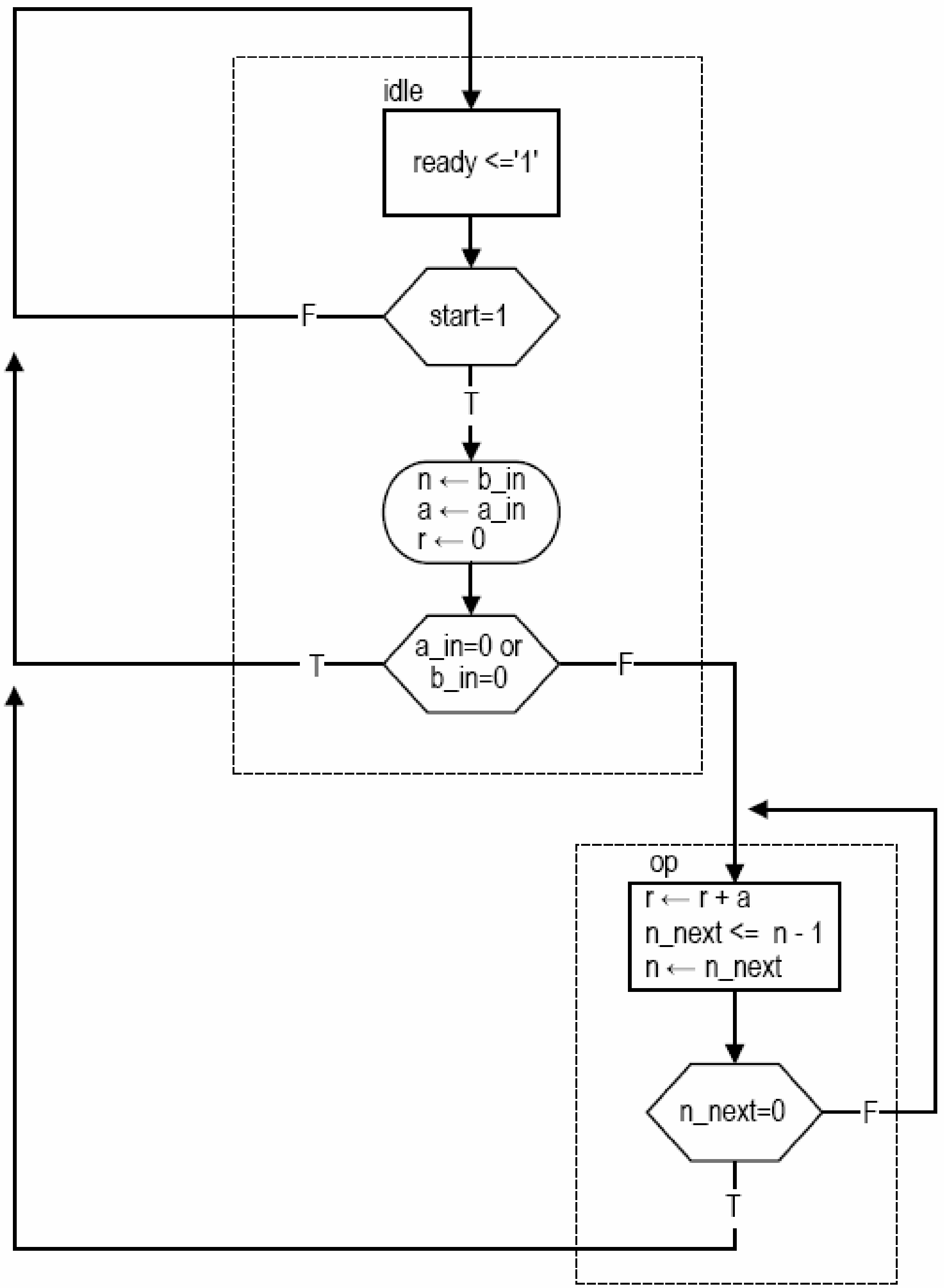


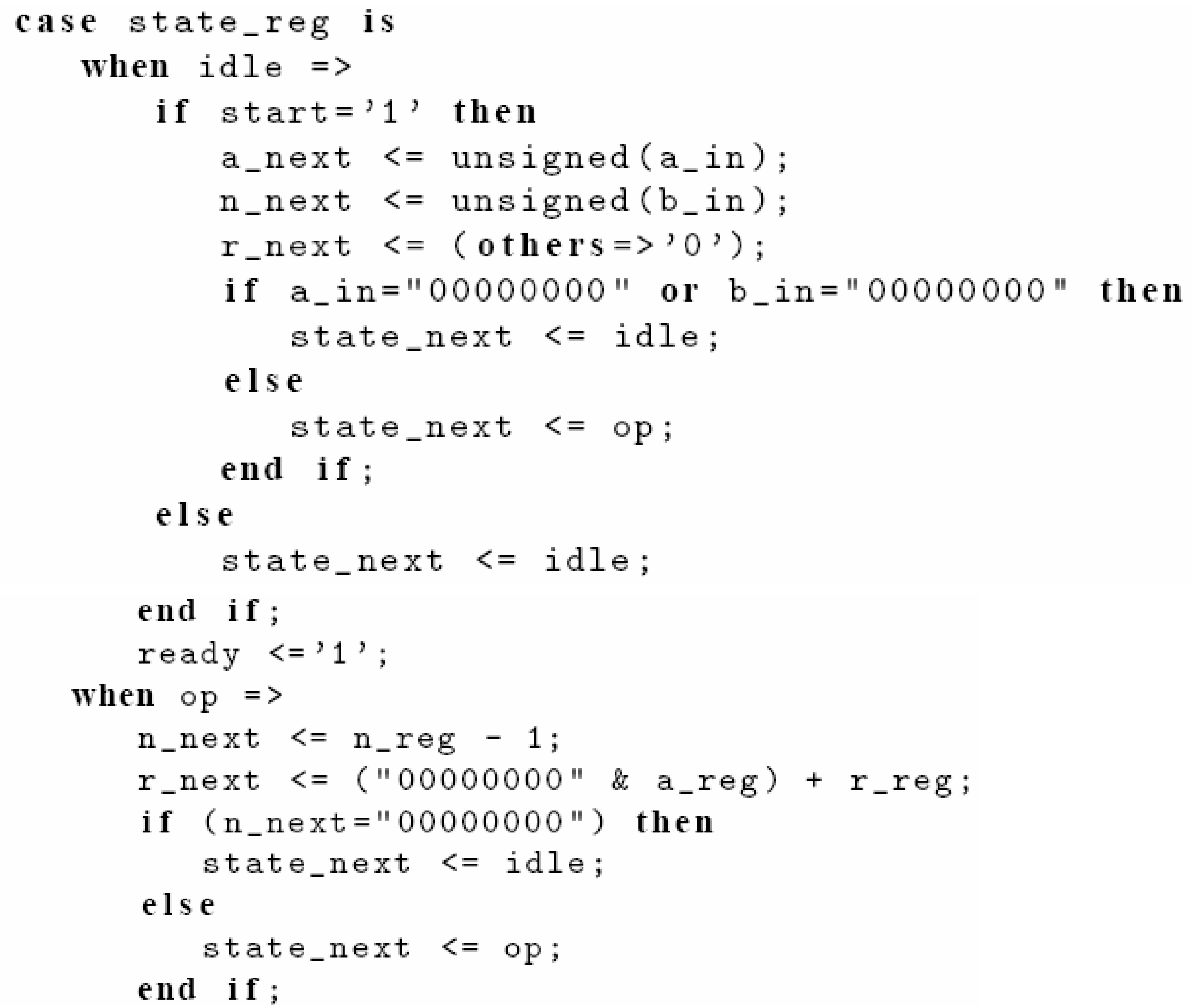
(a) ASMD block

(b) Conceptual block diagram

- Mealy control signal for multiplier
  - load and ab0 states perform no computation
  - Mealy control can be used to eliminate ab0 and load states

- r, n, b register loaded using Mealy signal



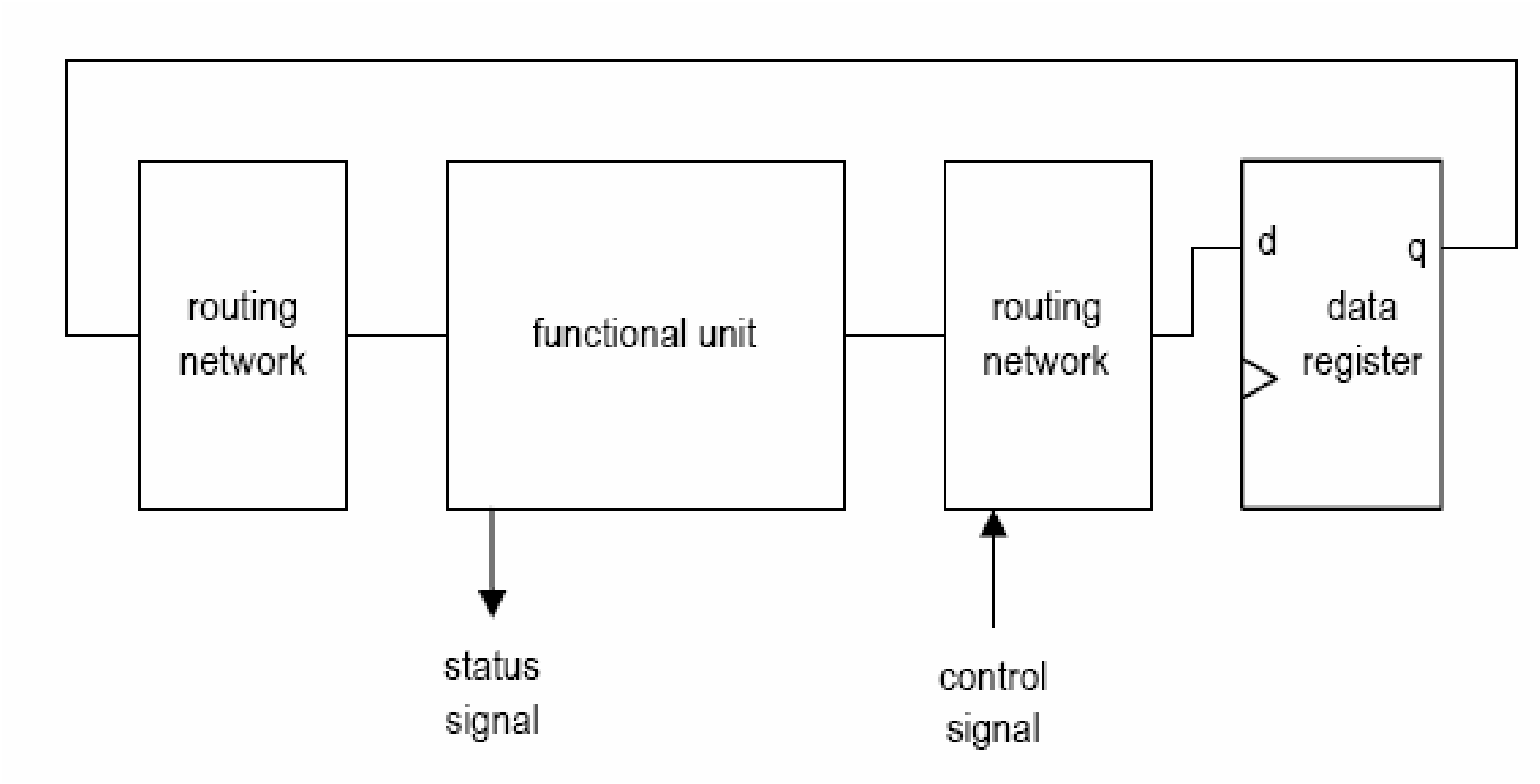


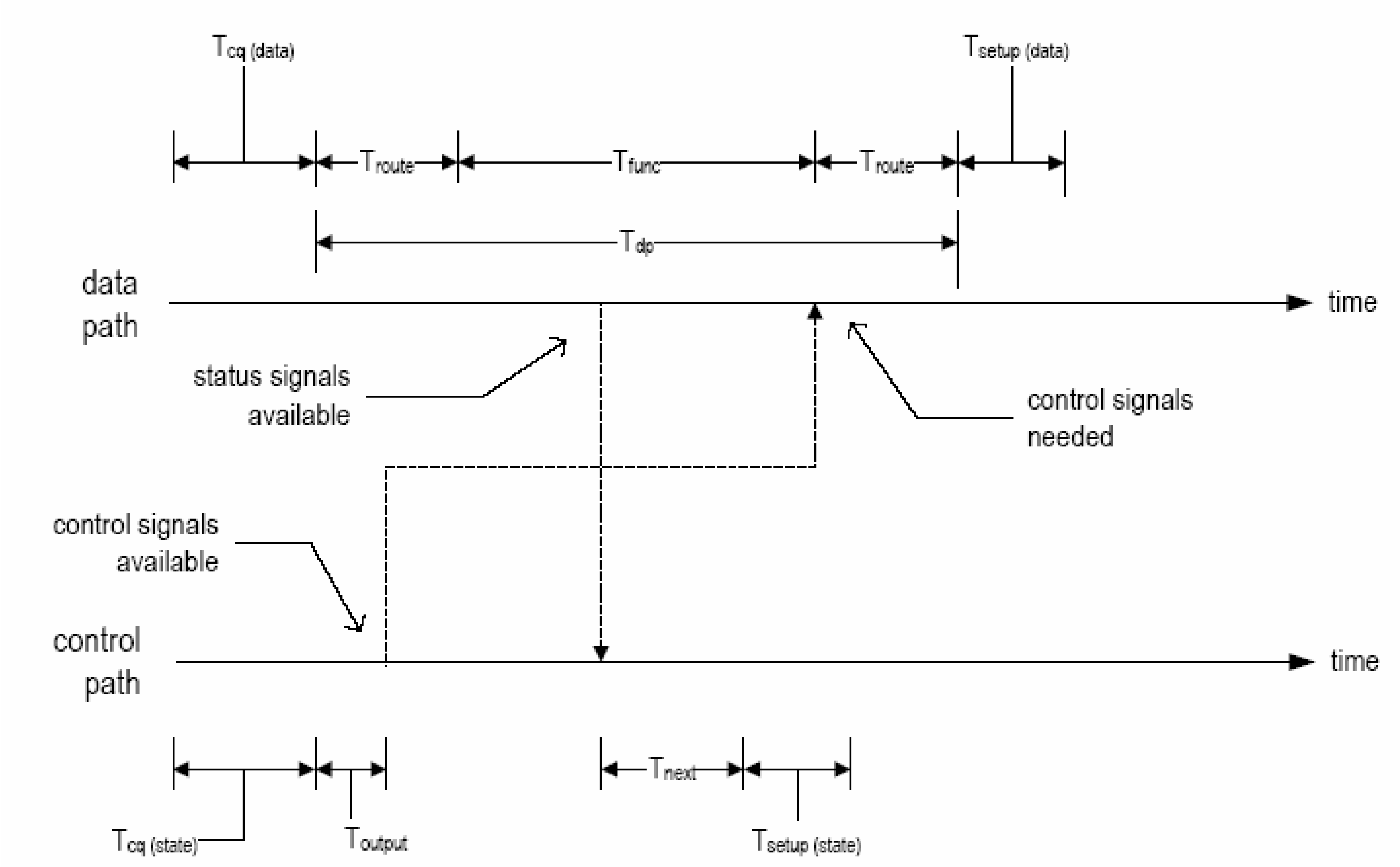
```
case state_reg is
   when idle =>
      if start='1' then
         a_next <= unsigned(a_in);
         n_next <= unsigned(b_in);
         r_next <= (others=>'0');
         if a_in="00000000" or b_in="00000000" then
            state_next <= idle;
         else
            state_next <= op;
         end if;
      else
         state_next <= idle;
      end if;
      ready <='1';
   when op =>
      n_next <= n_reg - 1;
      r_next <= ("00000000" & a_reg) + r_reg;
      if (n_next="00000000") then
         state_next <= idle;
      else
         state_next <= op;
      end if;
```

5. Clock rate and Performance of FSMD

- Maximal clock rate
  - More difficult to analyze because of two interactive loops
  - The boundary of the clock rate can be found

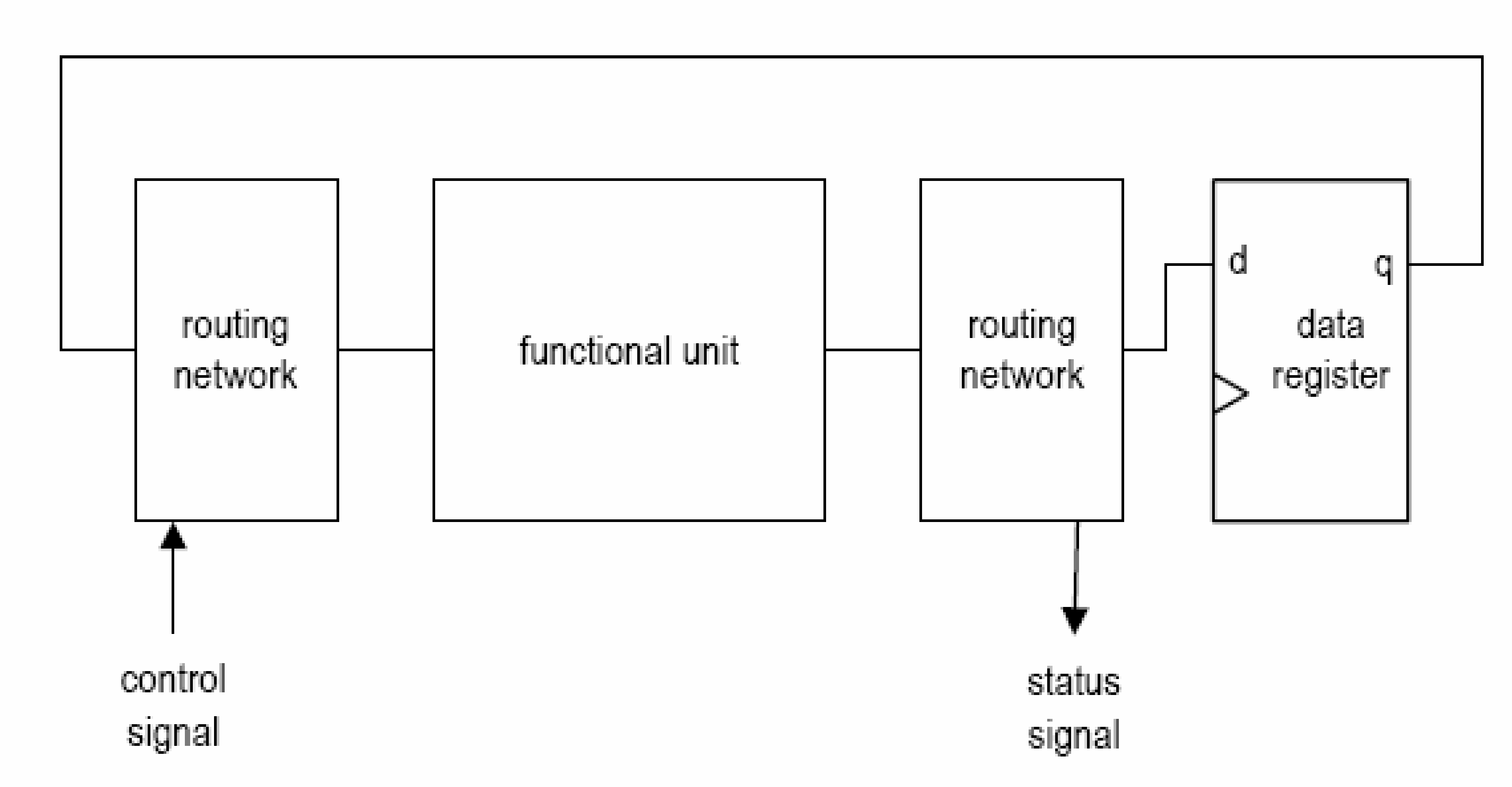
- Best-case scenario:
  - Control signals needed at late stage
  - Status signal available at early stage

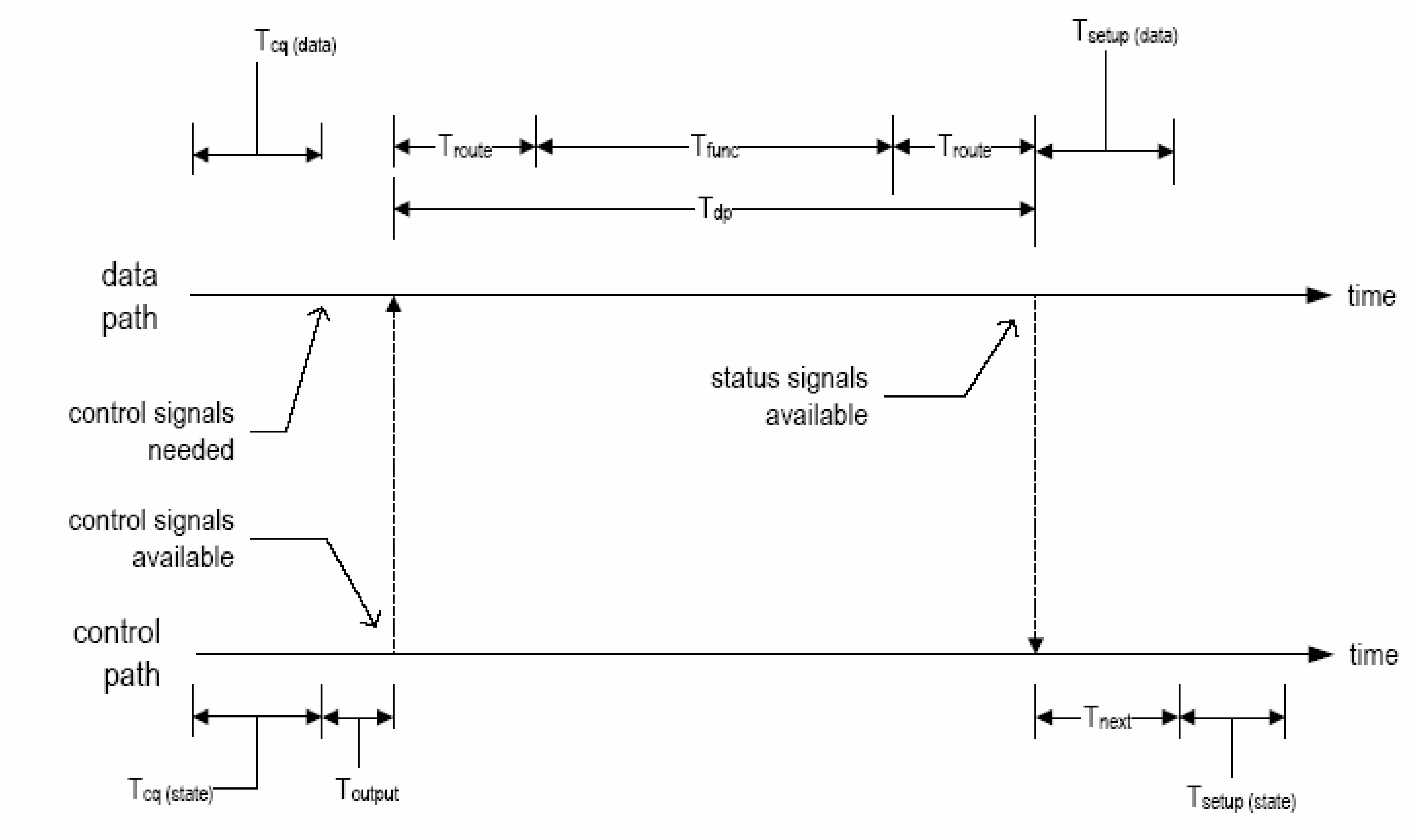




$$T_c = T_{cq(data)} + T_{dp} + T_{setup(data)}$$

- Best-case scenario:
  - – Control signals needed at early stage
  - – Status signal available at late stage





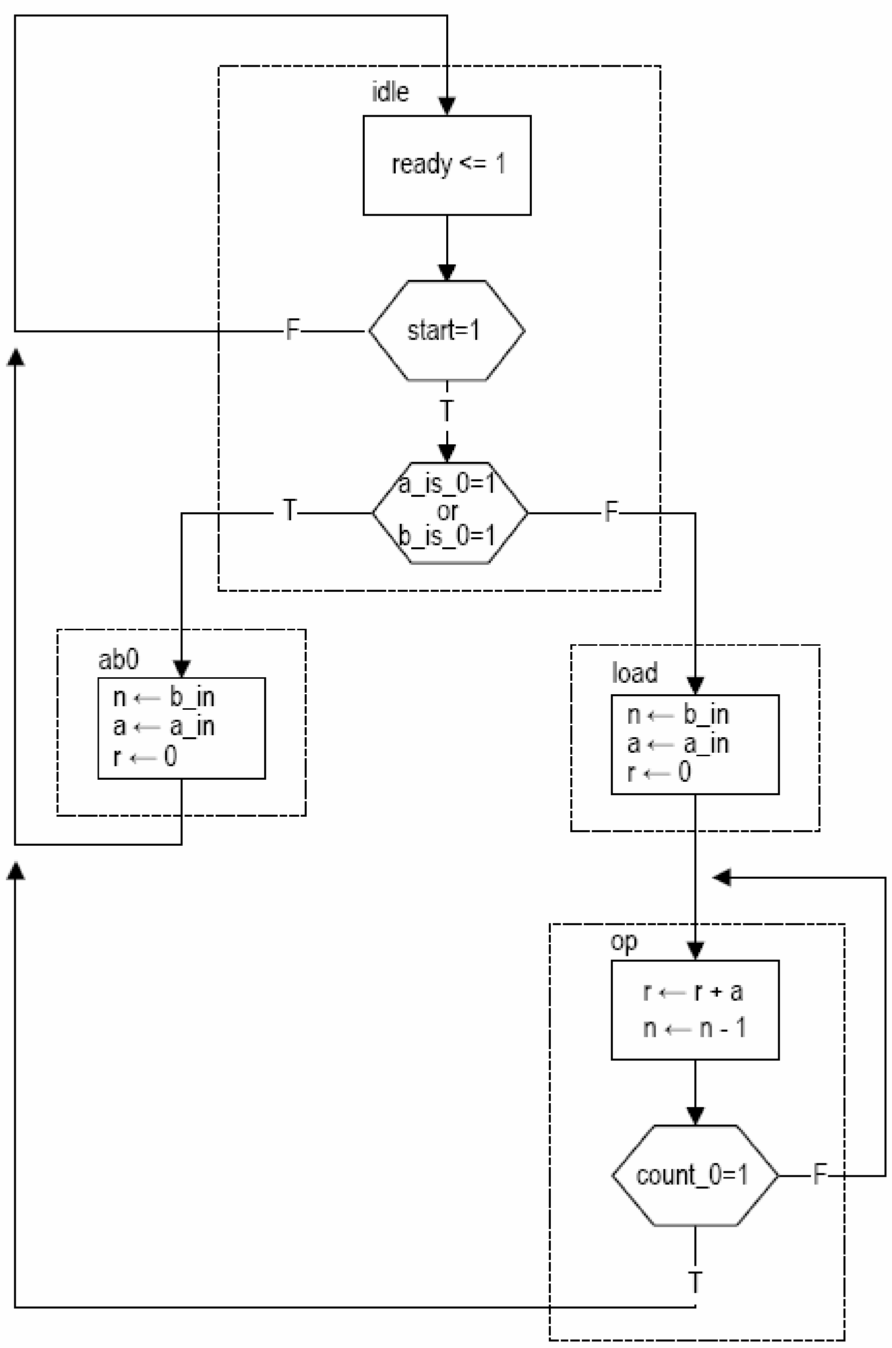
$$T_c = T_{cq(state)} + T_{output} + T_{dp} + T_{next} + T_{setup(state)}$$

$$T_{cq} + T_{dp} + T_{setup} \le T_c \le T_{cq} + T_{output} + T_{dp} + T_{next} + T_{setup}$$

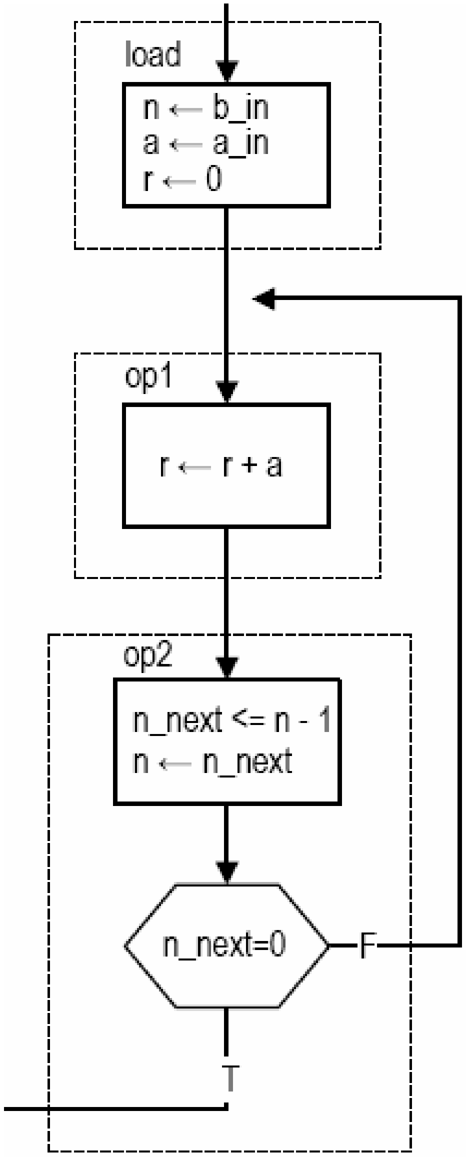
$$\frac{1}{T_{cq} + T_{output} + T_{dp} + T_{next} + T_{setup}} \le f \le \frac{1}{T_{cq} + T_{dp} + T_{setup}}$$

- Performance of FSMD
  - Tc: Clock period
  - K: # clock cycles to compete the computation
  - Total time = K * Tc
- K determined by algorithm, input patterns etc.

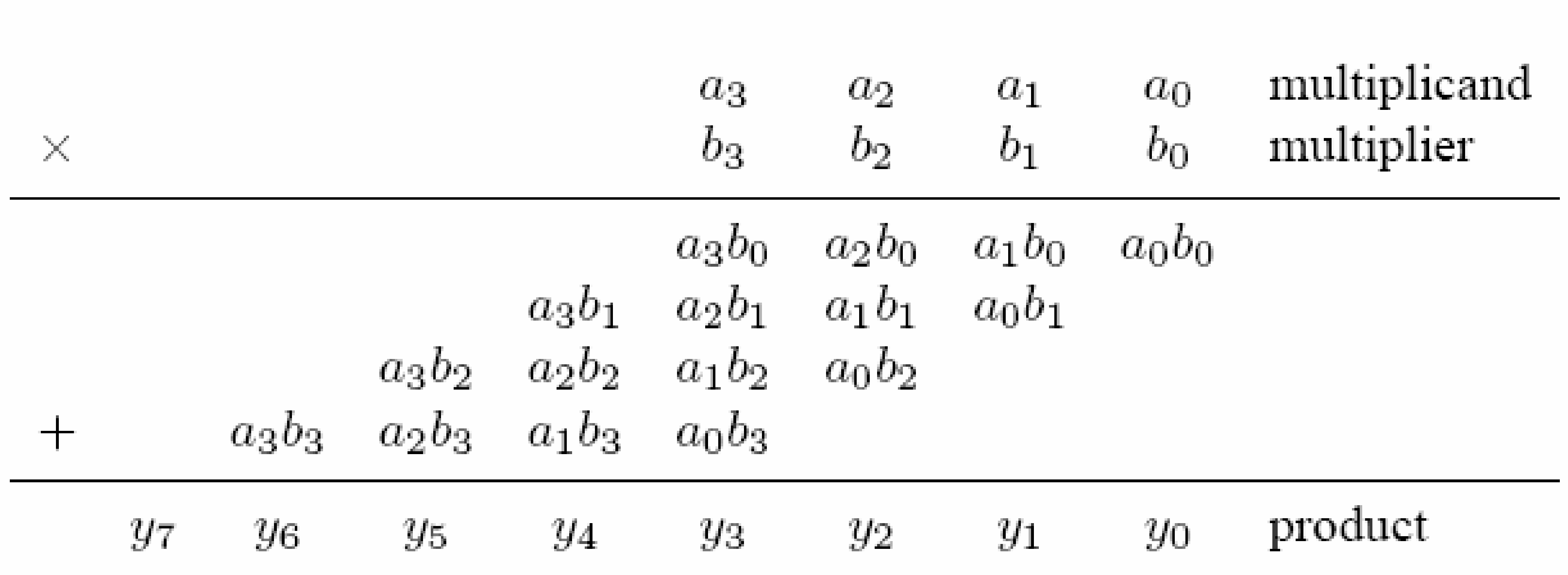
- 8-bit input
  - Best: b=0, K=2
  - Worst: b=255, K=257
- N-bit input:
  - Worst:
    K = 2+(2^n-1)



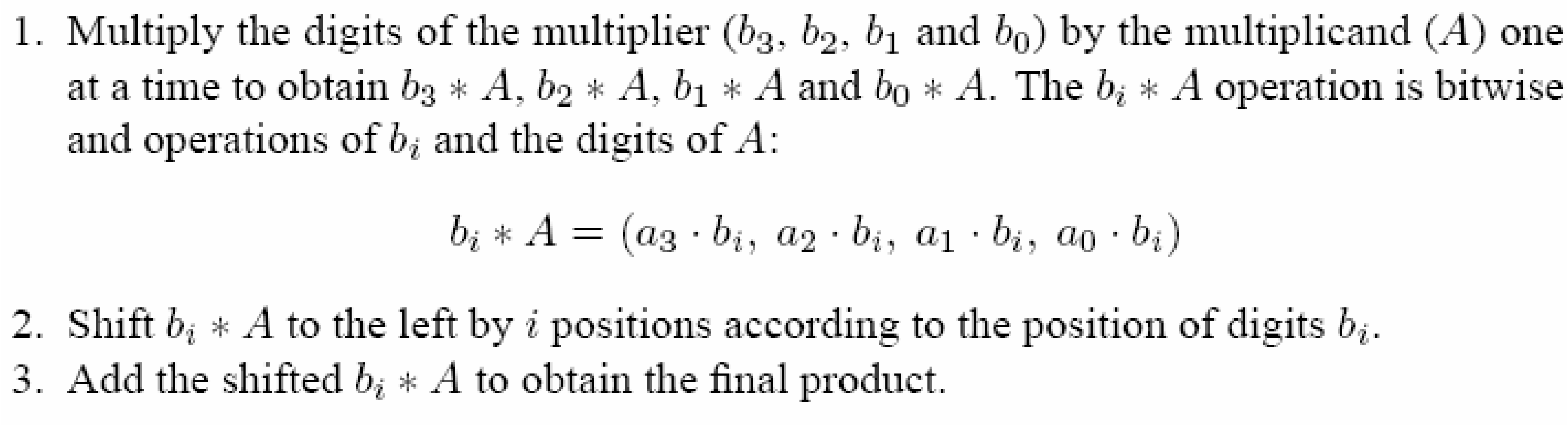
- 8-bit input
  - Best: b=0, K=2
  - Worst: b=255, K=2 + 255*2
- N-bit input:
  - Worst: $K=2+2*(2^n-1)$



6. Sequential add-and-shift multiplier



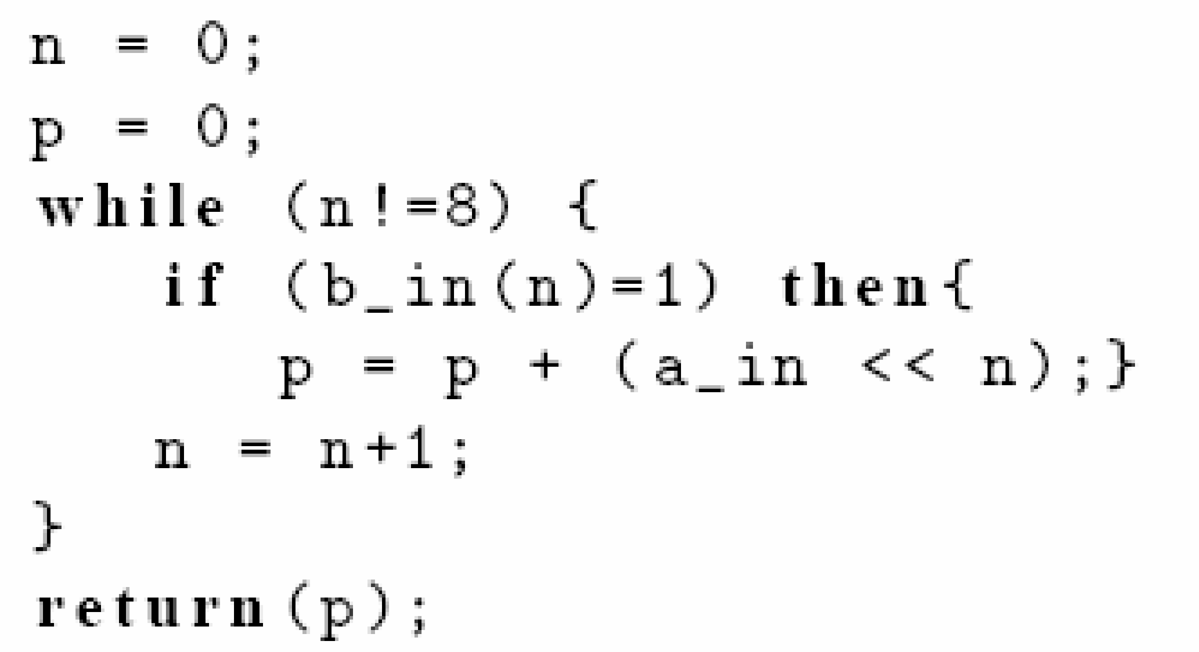
$$\begin{array}{cccccccccl}
 & & & & & a_3 & a_2 & a_1 & a_0 & \text{multiplicand} \\
\times & & & & & b_3 & b_2 & b_1 & b_0 & \text{multiplier} \\
\hline
 & & & & & a_3b_0 & a_2b_0 & a_1b_0 & a_0b_0 & \\
 & & & & a_3b_1 & a_2b_1 & a_1b_1 & a_0b_1 & & \\
 & & & a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 & & & \\
+ & & a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 & & & & \\
\hline
 & y_7 & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 & \text{product}
\end{array}$$



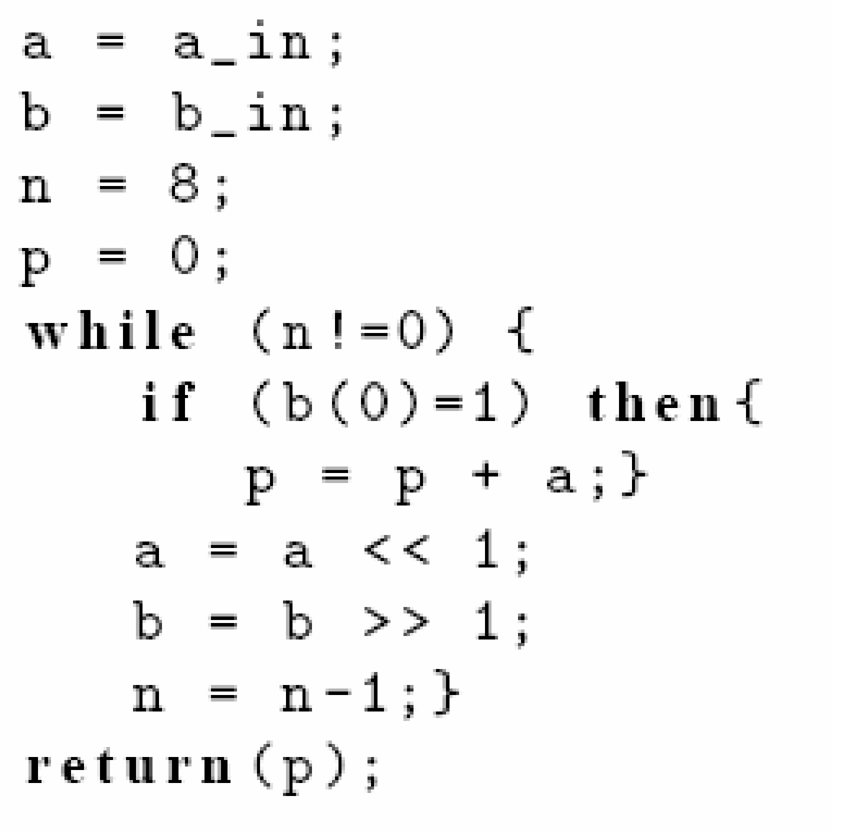
1. Multiply the digits of the multiplier (b_3, b_2, b_1 and b_0) by the multiplicand (A) one at a time to obtain $b_3 * A$, $b_2 * A$, $b_1 * A$ and $b_0 * A$. The $b_i * A$ operation is bitwise and operations of b_i and the digits of A:

$$b_i * A = (a_3 \cdot b_i,\ a_2 \cdot b_i,\ a_1 \cdot b_i,\ a_0 \cdot b_i)$$

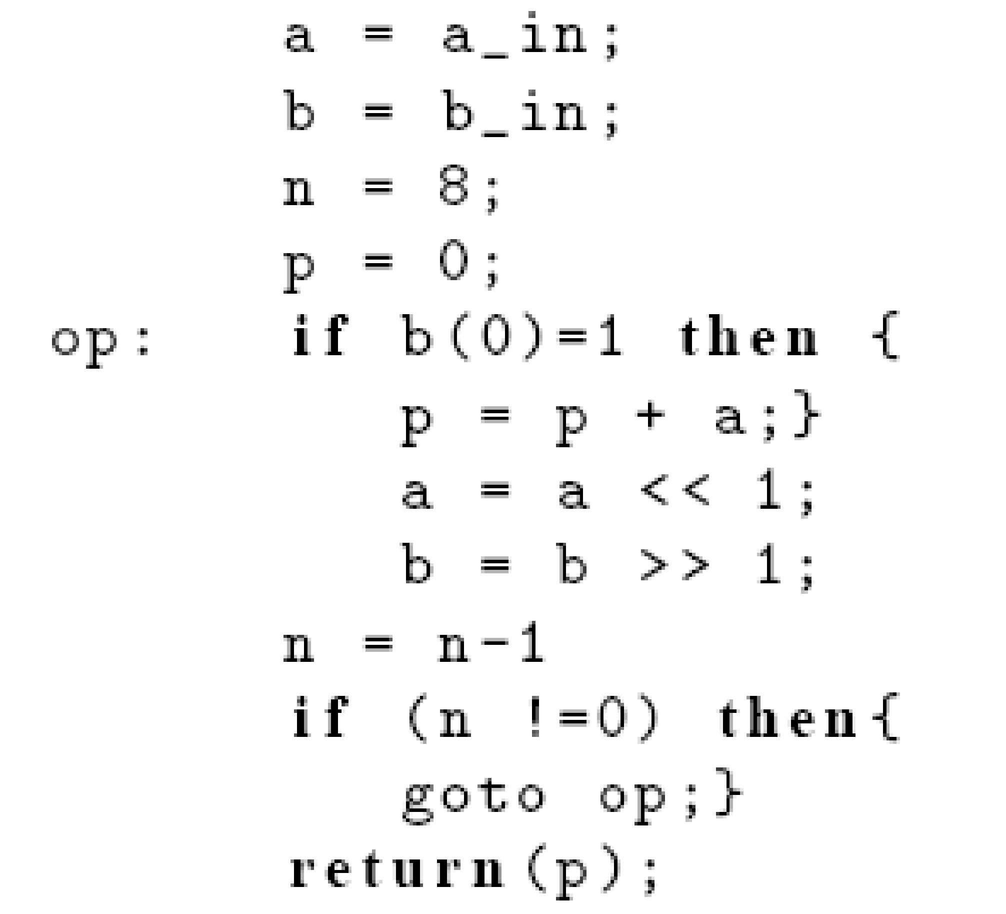
2. Shift $b_i * A$ to the left by i positions according to the position of digits b_i.
3. Add the shifted $b_i * A$ to obtain the final product.



```
n = 0;
p = 0;
while (n!=8) {
    if (b_in(n)=1) then{
        p = p + (a_in << n);}
    n = n+1;
}
return(p);
```

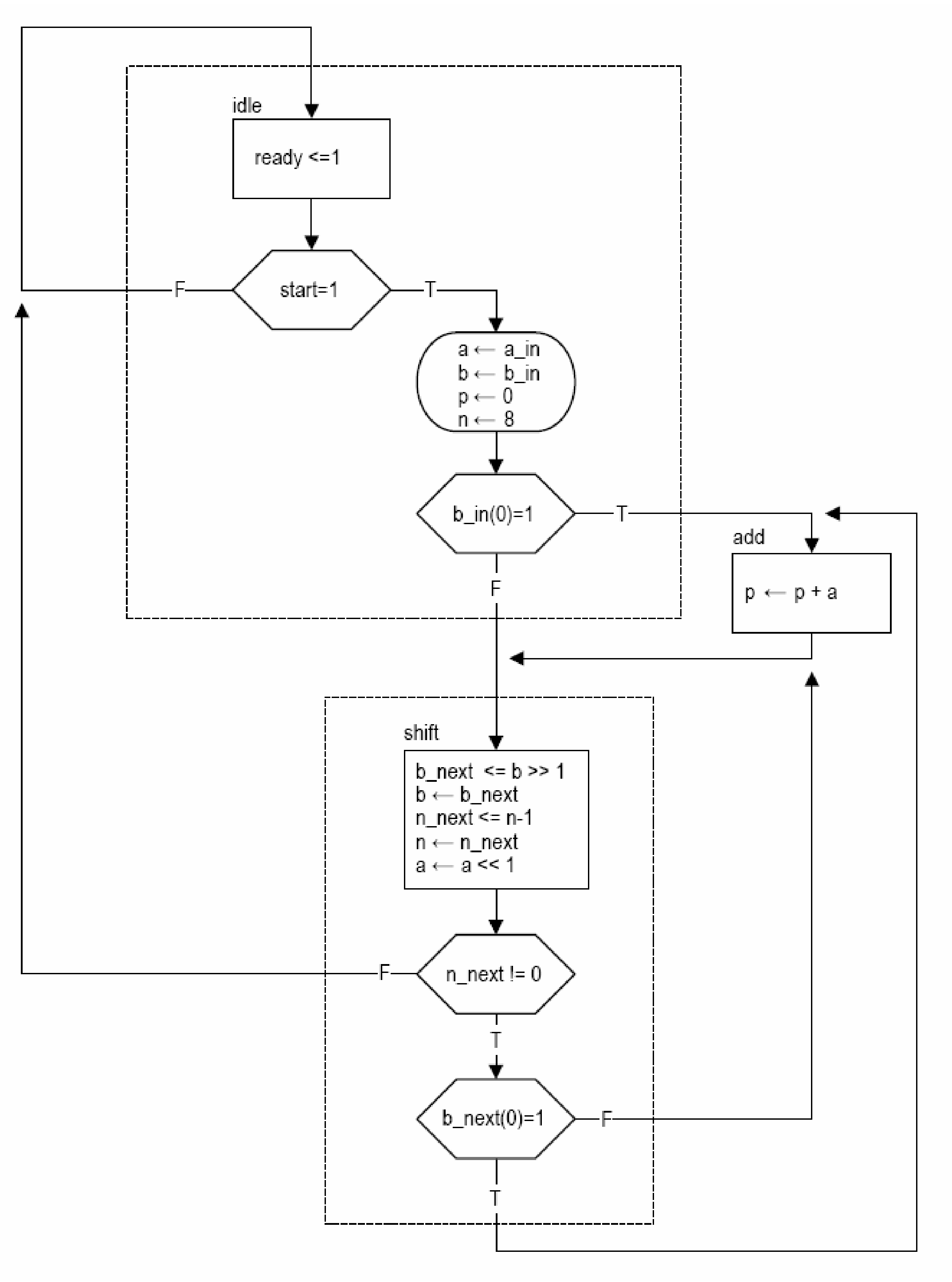


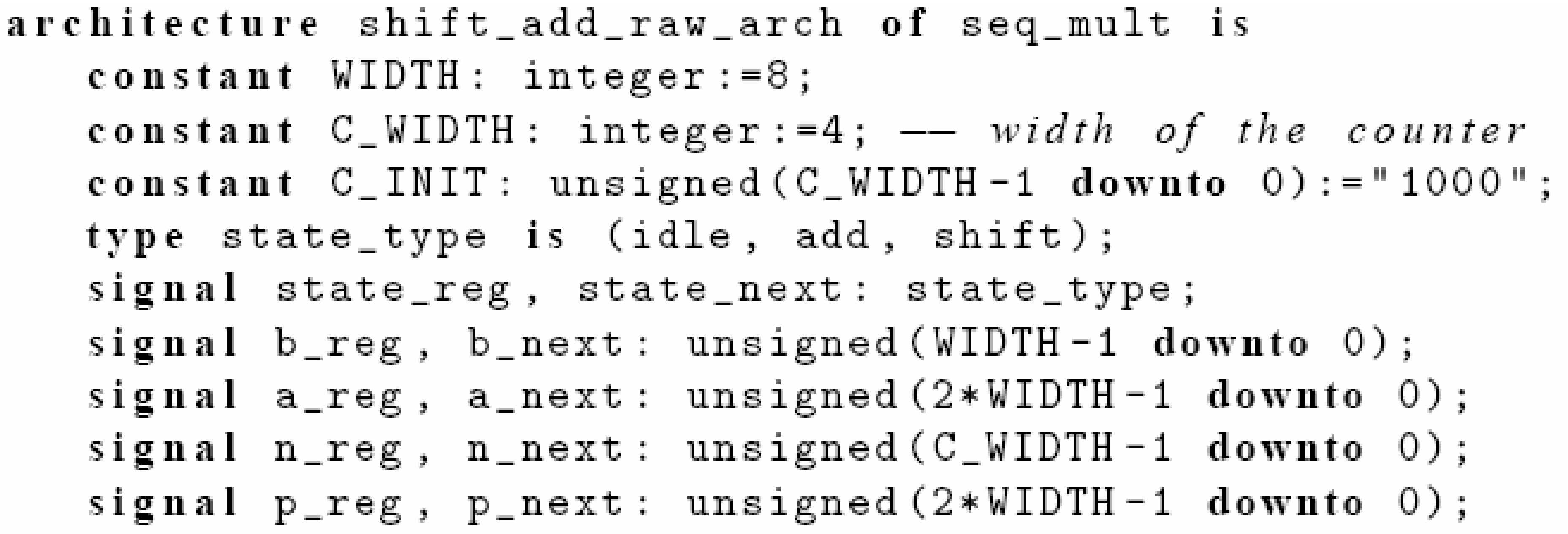
```
a = a_in;
b = b_in;
n = 8;
p = 0;
while (n!=0) {
    if (b(0)=1) then{
        p = p + a;}
    a = a << 1;
    b = b >> 1;
    n = n-1;}
return(p);
```



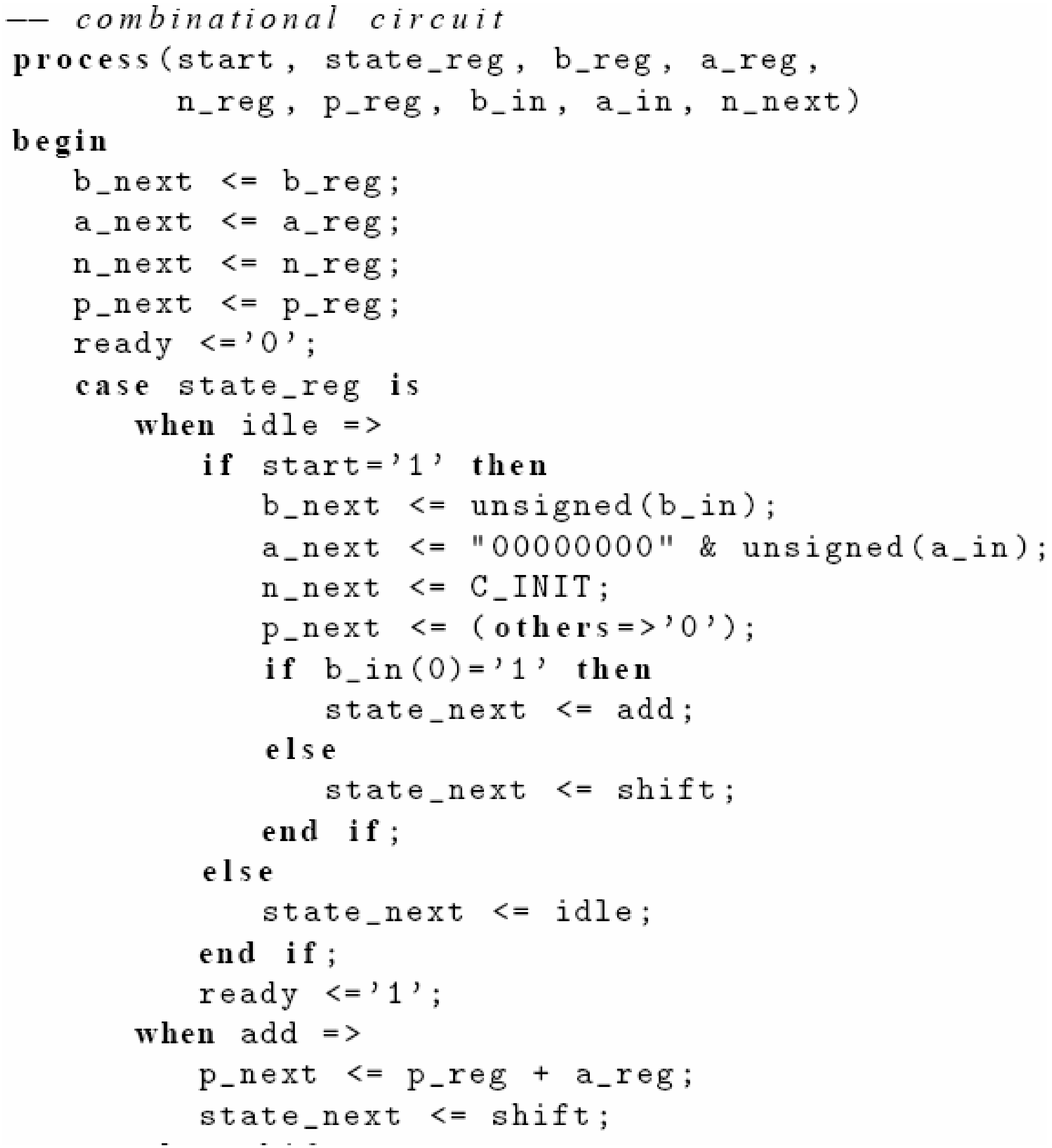
```
        a = a_in;
        b = b_in;
        n = 8;
        p = 0;
op:     if b(0)=1 then {
           p = p + a;}
           a = a << 1;
           b = b >> 1;
        n = n-1
        if (n !=0) then{
           goto op;}
        return(p);
```

- Note the use of b_next and n_next
- a<<1 and b>>1 require no logic
- 8-bit input
  - Best: b=0, K = 1 + 8
  - Worst: b=255, K = 1 + 8*2
- N-bit input:
  - Worst: K=2+2*n

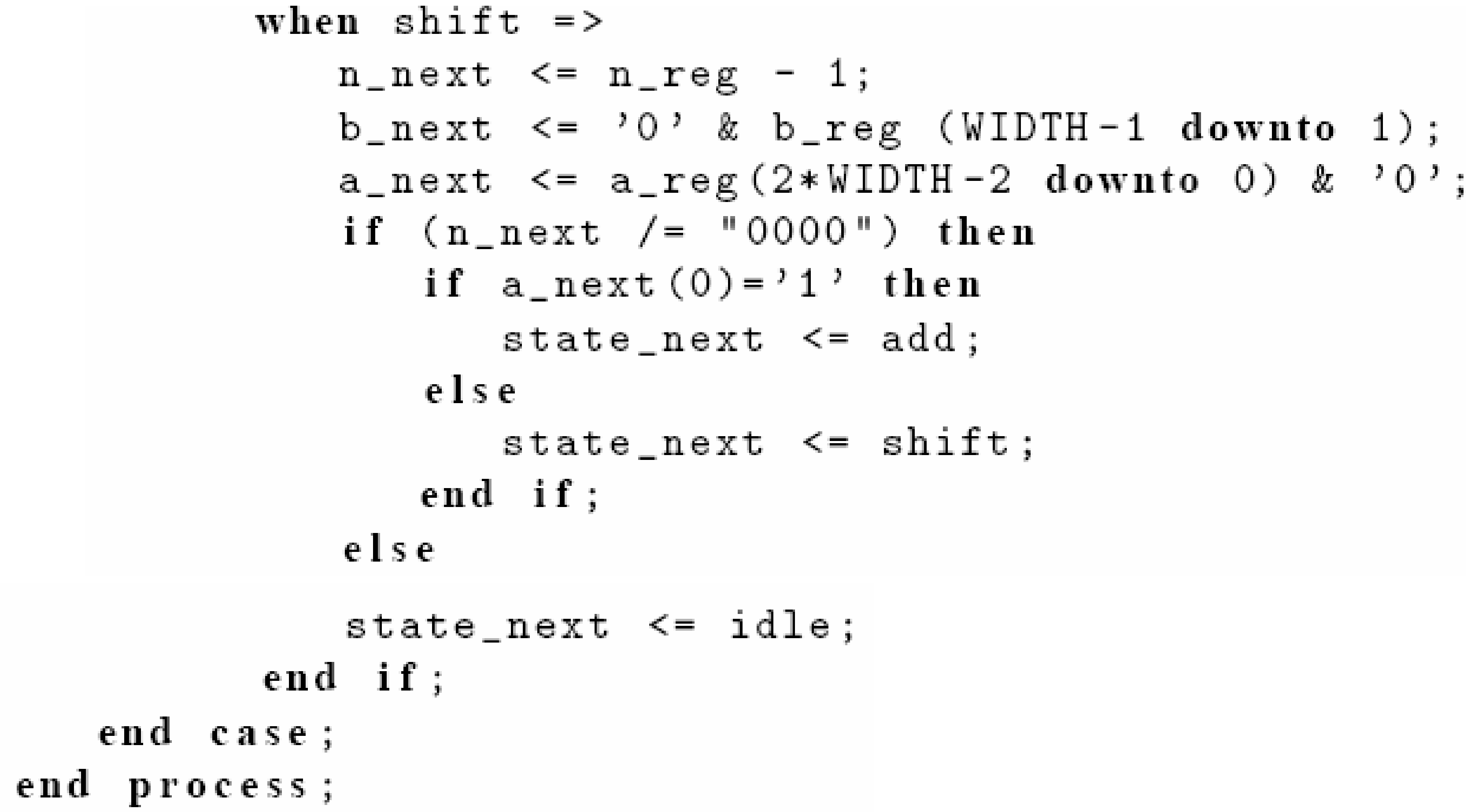




```
architecture shift_add_raw_arch of seq_mult is
   constant WIDTH: integer:=8;
   constant C_WIDTH: integer:=4; -- width of the counter
   constant C_INIT: unsigned(C_WIDTH-1 downto 0):="1000";
   type state_type is (idle, add, shift);
   signal state_reg, state_next: state_type;
   signal b_reg, b_next: unsigned(WIDTH-1 downto 0);
   signal a_reg, a_next: unsigned(2*WIDTH-1 downto 0);
   signal n_reg, n_next: unsigned(C_WIDTH-1 downto 0);
   signal p_reg, p_next: unsigned(2*WIDTH-1 downto 0);
```

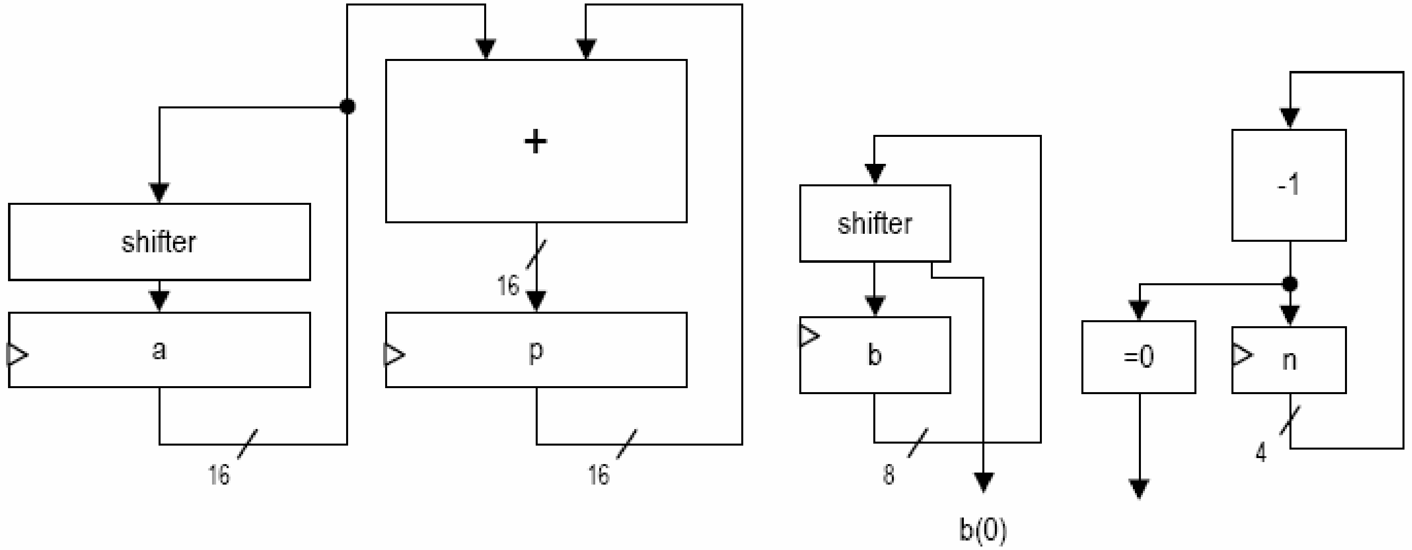


```
-- combinational circuit
process(start, state_reg, b_reg, a_reg,
        n_reg, p_reg, b_in, a_in, n_next)
begin
   b_next <= b_reg;
   a_next <= a_reg;
   n_next <= n_reg;
   p_next <= p_reg;
   ready <='0';
   case state_reg is
      when idle =>
         if start='1' then
            b_next <= unsigned(b_in);
            a_next <= "00000000" & unsigned(a_in);
            n_next <= C_INIT;
            p_next <= (others=>'0');
            if b_in(0)='1' then
               state_next <= add;
            else
               state_next <= shift;
            end if;
         else
            state_next <= idle;
         end if;
         ready <='1';
      when add =>
         p_next <= p_reg + a_reg;
         state_next <= shift;
```

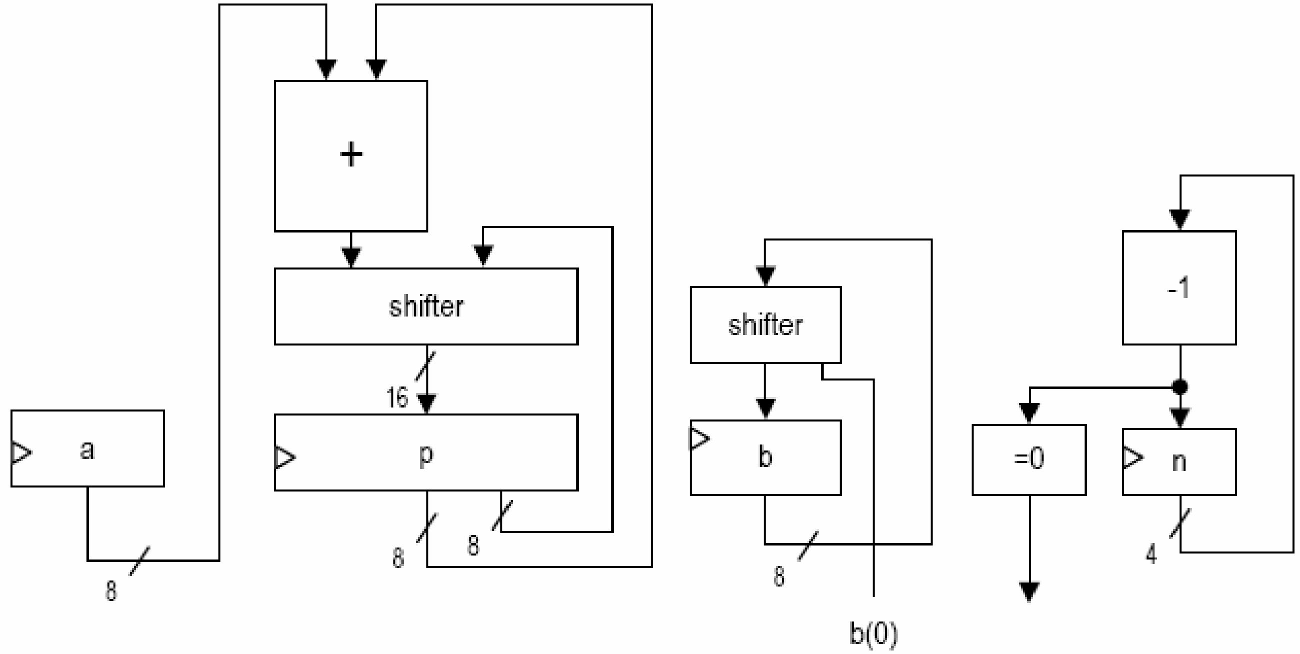


```
         when shift =>
            n_next <= n_reg - 1;
            b_next <= '0' & b_reg (WIDTH-1 downto 1);
            a_next <= a_reg(2*WIDTH-2 downto 0) & '0';
            if (n_next /= "0000") then
               if a_next(0)='1' then
                  state_next <= add;
               else
                  state_next <= shift;
               end if;
            else
            state_next <= idle;
         end if;
   end case;
end process;
```

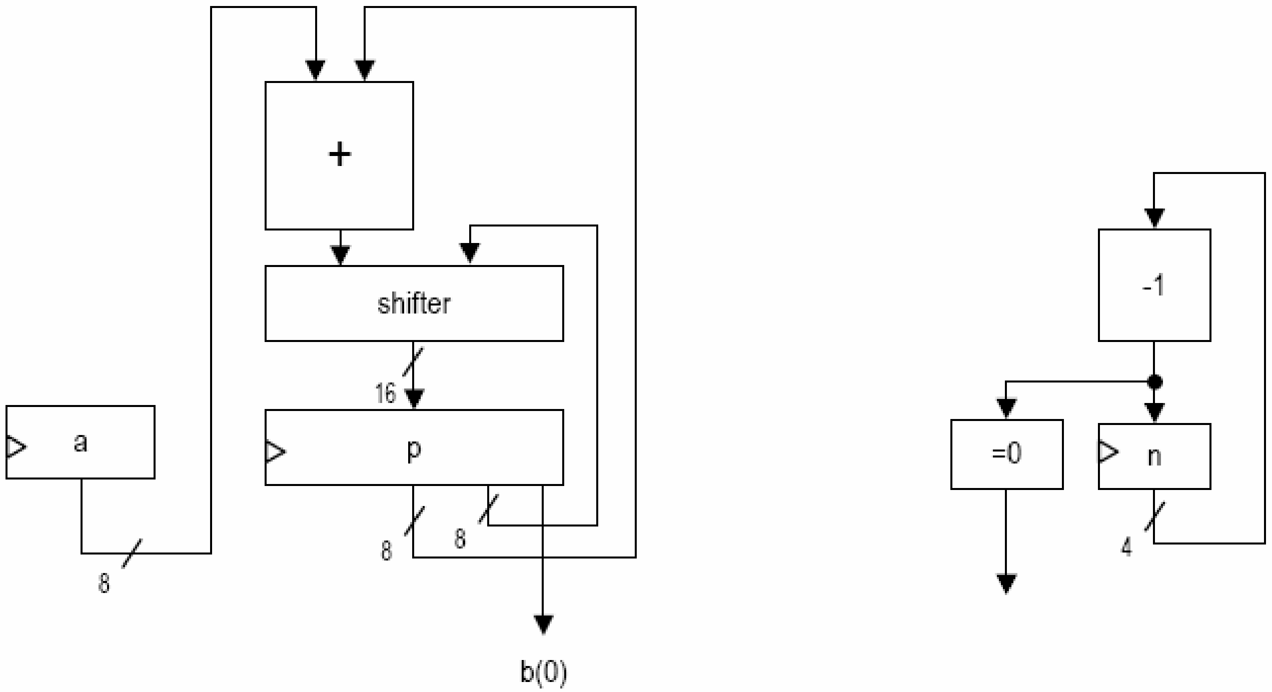
- Refinement
  - – No major computation done in the shift state: the add and shift states can be merged
  - – Data path can be simplified:
    - Replace 2n-bit adder with (n+1)-bit adder
    - Reduce the a register from 2n bits to n bits
    - Use the lower part of the p register to store B and eliminate the b register



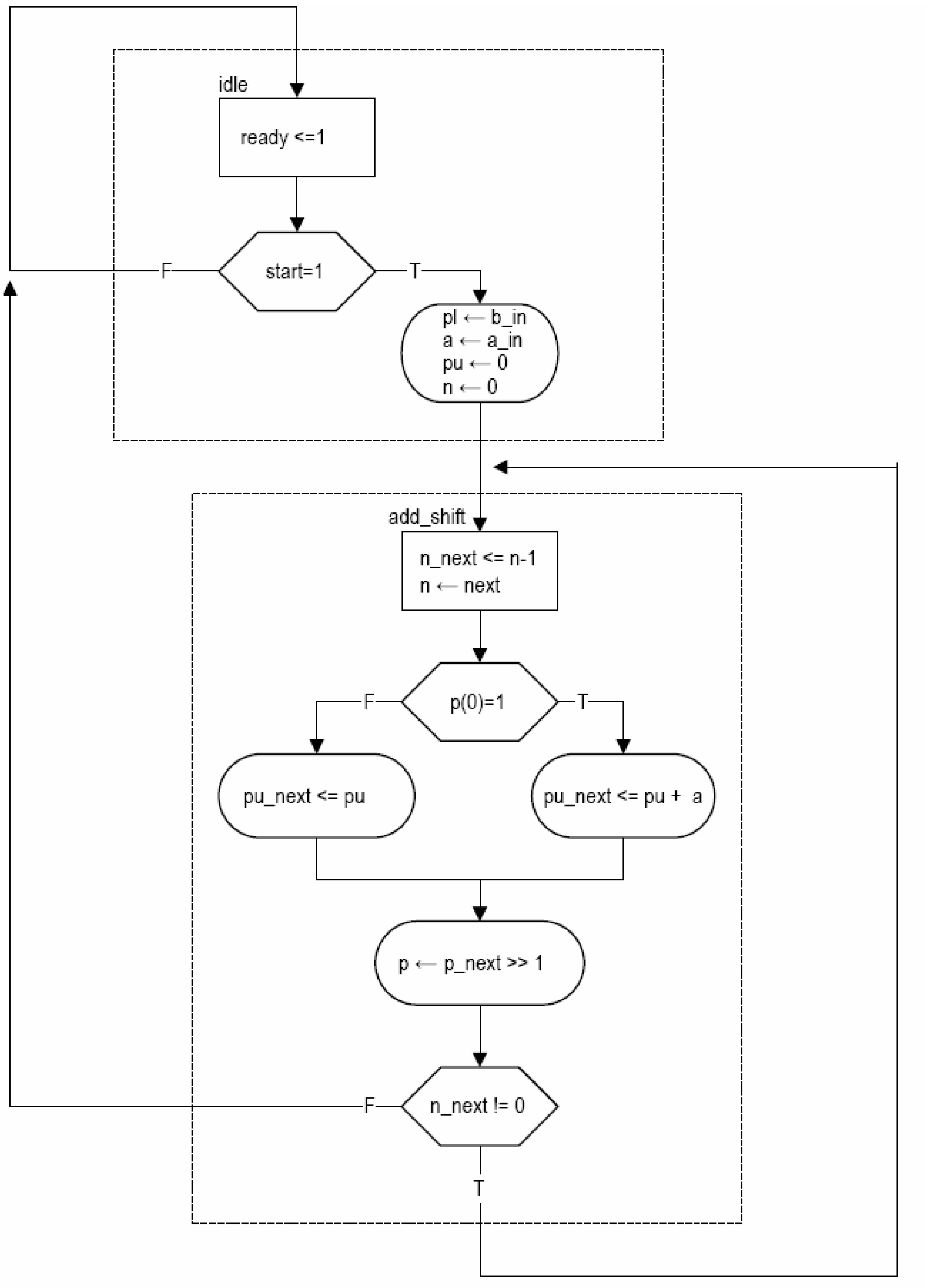
(a) Initial design

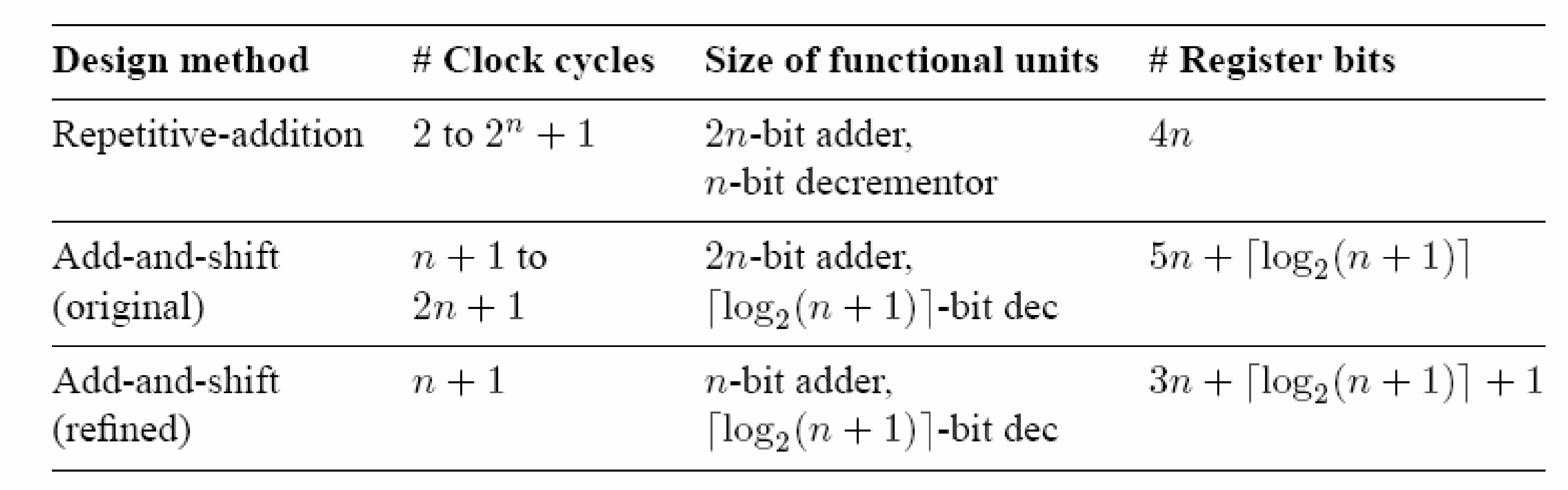


(b) "Shifting p register" design



(c) Final design


idle
ready <=1
start=1
F
T
pl ← b_in
a ← a_in
pu ← 0
n ← 0
add_shift
n_next <= n-1
n ← next
p(0)=1
F
T
pu_next <= pu
pu_next <= pu + a
p ← p_next >> 1
n_next != 0
F
T



Design method	# Clock cycles	Size of functional units	# Register bits
Repetitive-addition	2 to $2^n + 1$	$2n$-bit adder, n-bit decrementor	$4n$
Add-and-shift (original)	$n + 1$ to $2n + 1$	$2n$-bit adder, $\lceil \log_2(n+1) \rceil$-bit dec	$5n + \lceil \log_2(n+1) \rceil$
Add-and-shift (refined)	$n + 1$	n-bit adder, $\lceil \log_2(n+1) \rceil$-bit dec	$3n + \lceil \log_2(n+1) \rceil + 1$